NLO W hadroproduction matched with shower: the POWHEG method

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OUTLINE

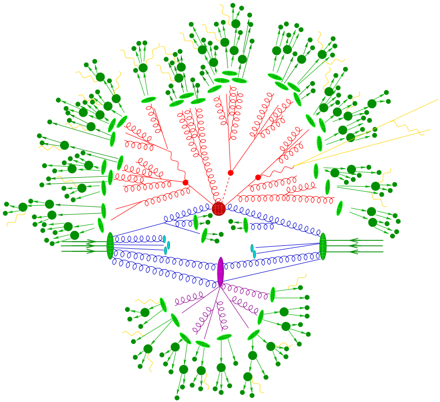
- (Quick) introduction
- LO vs. NLO vs. Standard Monte Carlo's: need of higher accuracy
- Merging of NLO and Parton Showers: the `POWHEG` method
- `POWHEG` results for W hadroproduction
- Conclusions and future prospects

INTRODUCTION (1/3)

Our way of describing (high-energy) hadronic collisions is based on the parton model and is well summarized by this picture:

Main stages:

- beam of hadrons = beam of partons (parton model)
- radiation off incoming partons
- primary hard scattering ($\mu \approx Q \gg \Lambda_{QCD}$)
- radiation off outgoing partons ($Q > \mu > \Lambda_{QCD}$)
- hadronization ($\mu \approx \Lambda_{QCD}$)
- secondary scatterings and/or underlying event



- hard scattering: pQCD, EW, BSM
- all the rest: radiation off charged particles (QCD&EW showers) + hadronization

Monte Carlo Event Generators are computer codes able to simulate all these stages...

INTRODUCTION (2/3)

... and to produce as output a detailed description of the generated 'events' (momenta of all outgoing leptons and hadrons):

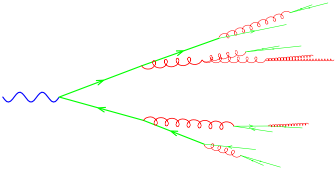
IHEP	ID	IDPDG	IST	MO1	MO2	DA1	DA2	P-X	P-Y	P-Z	ENERGY
31	NU_E	12	1	29	22	0	0	60.53	37.24	-1185.0	1187.1
32	E+	-11	1	30	22	0	0	-22.80	2.59	-232.4	233.6
148	K+	321	1	109	9	0	0	-1.66	1.26	1.3	2.5
151	PI0	111	1	111	9	0	0	-0.01	0.05	11.4	11.4
152	PI+	211	1	111	9	0	0	-0.19	-0.13	2.0	2.0
153	PI-	-211	1	112	9	0	0	0.84	-1.07	1626.0	1626.0
154	K+	321	1	112	9	0	0	0.48	-0.63	945.7	945.7
155	PI0	111	1	113	9	0	0	-0.37	-1.16	64.8	64.8
156	PI-	-211	1	113	9	0	0	-0.20	-0.02	3.1	3.1
158	PI0	111	1	114	9	0	0	-0.17	-0.11	0.2	0.3
159	PI0	111	1	115	18	0	0	0.18	-0.74	-267.8	267.8
160	PI-	-211	1	115	18	0	0	-0.21	-0.13	-259.4	259.4
161	N	2112	1	116	23	0	0	-8.45	-27.55	-394.6	395.7
162	NBAR	-2112	1	116	23	0	0	-2.49	-11.05	-154.0	154.4
163	PI0	111	1	117	23	0	0	-0.45	-2.04	-26.6	26.6
164	PI0	111	1	117	23	0	0	0.00	-3.70	-56.0	56.1
167	K+	321	1	119	23	0	0	-0.40	-0.19	-8.1	8.1
186	PBAR	-2212	1	130	9	0	0	0.10	0.17	-0.3	1.0

that are used as inputs by high-energy experimentalists to study analysis strategies.

- Traditional Monte Carlo's like HERWIG or PYTHIA have LO accuracy in the hard scattering and resum Leading Logarithms in the shower
- Inclusion of higher-order corrections in Monte Carlo generators is one of the main recent development in this field

This talk will deal only with QCD corrections.

INTRODUCTION (3/3)



Collinear (and soft) splitting processes in the initial and final state are strongly enhanced. This is due to the fact that, in perturbation theory, propagators can go almost on-shell:

$$\mathrm{q}(Q) \rightarrow \mathrm{q}(k) + \mathrm{g}(l)$$

$$\mathcal{M} \sim \frac{1}{(k+l)^2} = \frac{1}{2E_q E_g (1 - \cos\theta_{\mathrm{qg}})}$$

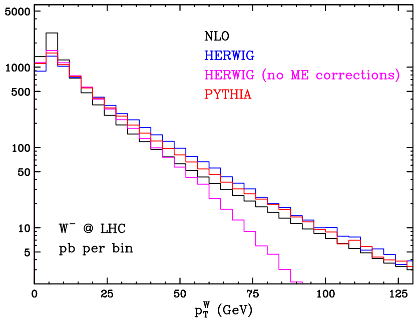
- When compared with the Born cross section, this enhancement is of order 1 $\rightarrow$ need of include it
- The whole analytic structure would be arbitrarly complicated $\rightarrow$ approximation $\rightarrow$ resummation of (some classes of) Logs
- Virtual corrections has to be included at the same level of accuracy $\rightarrow$ Sudakov form factors

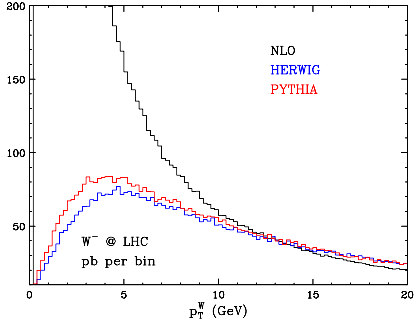
$\Rightarrow$ Shower algorithms

- Description of a hard collision up to distances of order $1/\Lambda_{QCD}$ (in the domain of *p*QCD)
- At larger distances, *p*QCD breaks down: need of models of hadronization, that should be process independent.

NLO vs. SMC's (LO + Parton Shower)

W^- @ LHC, no K-factors



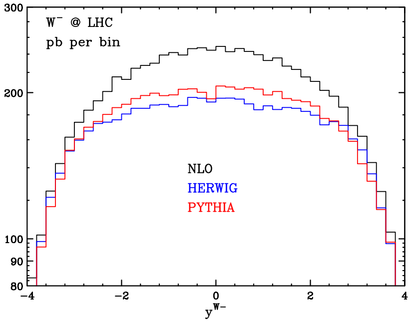


- NLO alone fails at low p_T (no Sudakov suppression of small p_T radiation)
- HERWIG (without ME corrections) wrong in shape at large p_T

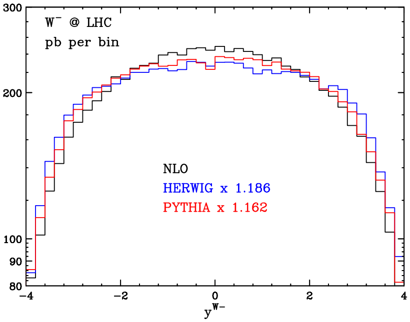
NLO vs. SMC's (LO + Parton Shower)

W^- @ LHC

no K-factors



K-factors included



- HERWIG and PYTHIA are wrong in the overall normalization

$$\frac{\sigma_{\text{NLO}}}{\sigma_{\text{SMC}}} \simeq 1.15 - 1.19$$

NEED OF HIGHER ACCURACY

NLO

- accurate shapes at high-p_T
- total normalization accurate at order α_s
- reduced dependence on μ_R and μ_F (virtual corrections included)
- wrong shapes in small-p_T region
- not so easy to be used: non trivial numerical cancellations to produce a sensible plot
- description only at the parton level

SMC's (LO+shower)

- bad description of high-p_T emissions
- total normalization accurate only at LO
- Sudakov suppression of small p_T emissions
- can be used as black-box
- simulate events down to the hadron level $\rightarrow$ used by experimentalists to study detectors calibration and find strategies to isolate signals from backgrounds

$\Downarrow$

It is natural to try to merge the 2 approaches, keeping the good features of both.

- As one can argue, the main problem is to avoid the double-counting of emissions
- There are two known and tested ways to perform the merging: `MC@NLO` (Frixione, Webber 2001) and `POWHEG` (Nason 2004)

MERGING NLO AND PARTON SHOWER

- We need to look the formula for a NLO calculation and for the first branching of a LO Parton Shower.
- NLO cross section:

$$d\sigma_{NLO} = d\Phi_n \Big\{ B(\Phi_n) + V(\Phi_n) + [R(\Phi_{n+1}) - C(\Phi_{n+1})]\, d\Phi_r \Big\}$$

  where

$$d\Phi_{n+1} = d\Phi_n d\Phi_r \;,\; \Phi_r = \{t, z, \varphi\} \;,\; V(\Phi_n) = V_{div}(\Phi_n) + \int d\Phi_r C(\Phi_n, \Phi_r)$$

  and

$$\frac{R(\Phi_{n+1})}{B(\Phi_n)}\, d\Phi_r \to \left(\frac{\alpha_s}{2\pi} \frac{1}{t} P(z) \right) dt\, dz \text{ when } t \to 0 \qquad \text{coll. factorization}$$

- Due to the ordering in t, the SMC hard emission cross section is roughly given by the shower first emission contribution:

$$d\sigma_{SMC} = B(\Phi_n) d\Phi_n \left[\Delta(t_0) + \Delta(t) \frac{\alpha_s}{2\pi} \frac{1}{t} P(z) d\Phi_r \right]$$

$$\Delta(t) = \exp \left\{ - \int_t^{t_{max}} d\Phi_r \, \frac{\alpha_s}{2\pi} \frac{1}{t} P(z) \right\} \qquad \text{SMC Sudakov form factor}$$

THE POWHEG METHOD (1/2)

The key idea is to *modify* $d\sigma_{SMC}$ *in such a way that*, expanding in α_s, *one recovers the exact NLO cross section*.

- With the substitutions

$$B(\Phi_n) \quad \Rightarrow \quad \bar{B}(\Phi_n) = B(\Phi_n) + V(\Phi_n) + \int \left[R(\Phi_{n+1}) - C(\Phi_{n+1})\right] d\Phi_r$$

$$\Delta(t) \quad \Rightarrow \quad \Delta(k_T) = \exp\left\{ - \int_{k_T} \frac{R(\Phi_{n+1})}{B(\Phi_n)} d\Phi_r \right\} \qquad \text{POWHEG Sudakov}$$

one obtains the POWHEG master formula for the hardest emission:

$$d\sigma_{POW} = \bar{B}(\Phi_n)\, d\Phi_n \left\{ \Delta(k_T^{min}) + \Delta(k_T) \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} d\Phi_r \right\}$$

- To avoid double-counting, *subsequent emissions must be* p_T *vetoed !*
- At high k_T, $\Delta(k_T) \to 1$, so the NLO large k_T accuracy is preserved:

$$d\sigma_{POW} \simeq \bar{B} \times \frac{R}{B} d\Phi_{n+1} \approx R\, d\Phi_{n+1} \times \ (1 + \mathcal{O}(\alpha_s))$$

- At small k_T the POWHEG Sudakov reduces to the SMC one. Thus:

all features of SMC's are preserved when $k_T \to 0$

THE POWHEG METHOD (2/2)

Differences with MC@NLO:

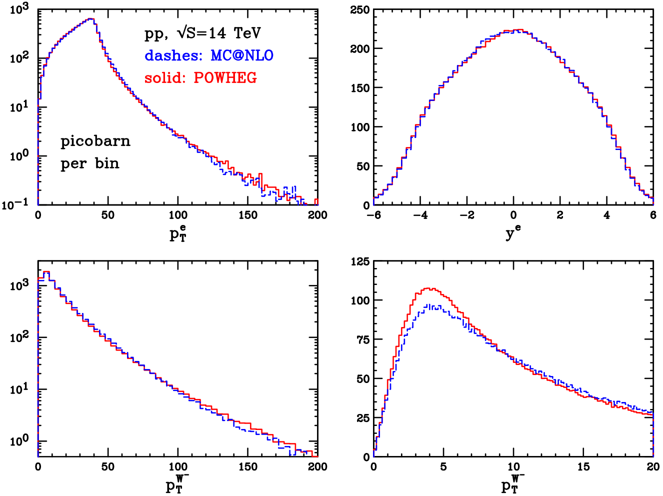
- Only positive weighted events are generated: $\bar{B} > 0$, because LO $>$ NLO
- The method is independent from the subsequent shower, while MC@NLO works only with the HERWIG shower
- We noticed that MC@NLO has problems in filling some Phase Space regions ("dips in jet rapidities"). The reason is that some events with high-k_T are generated with the HERWIG accuracy

Available POWHEG implementations:

- $pp \to ZZ$ [Nason and Ridolfi,JHEP 0608:077,2006]
- $pp \to Q\bar{Q}$, $Q = c, b, t$ with spin correlations in top decay [Frixione,Nason and Ridolfi,JHEP 0709:126,2007]
- $e^+e^- \to q\bar{q}$ [Latunde-Dada,Gieseke,Webber,JHEP 0702:051,2007]
  $e^+e^- \to t\bar{t}$ with top decay [Latunde-Dada, Eur.Phys.J.C58:543-554,2008]
- $pp \to Z, W$ with spin correlations [Alioli,Nason,Oleari,ER, JHEP 0807:060,2008] [Hamilton,Richardson and Tully, JHEP 0810:015,2008]
- $pp \to H$ via gluon fusion [Alioli,Nason,Oleari,ER, arXiv:0812.0578] , also in HERWIG++
- $pp \to W'$ [Latunde-Dada and Papaefstathiou, arXiv:0901.3685]

RESULTS: POWHEG VS. MC@NLO

W^- @ LHC, POWHEG interfaced to HERWIG

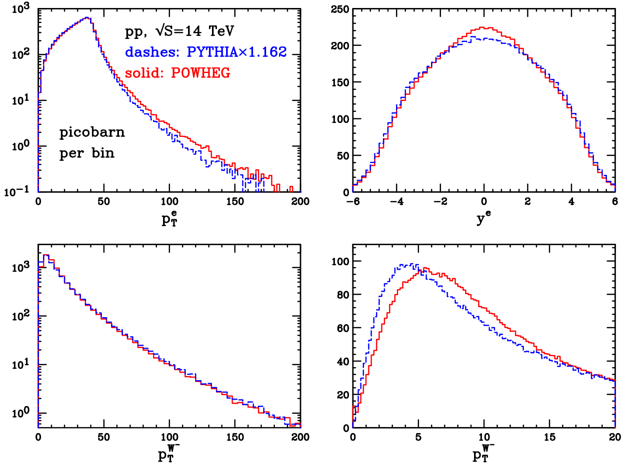


- Good agreement both at high and low p_T ;
- Similar results @TeV

[Alioli,Nason,Oleari,ER, JHEP 0807:060]

RESULTS: POWHEG VS. PYTHIA

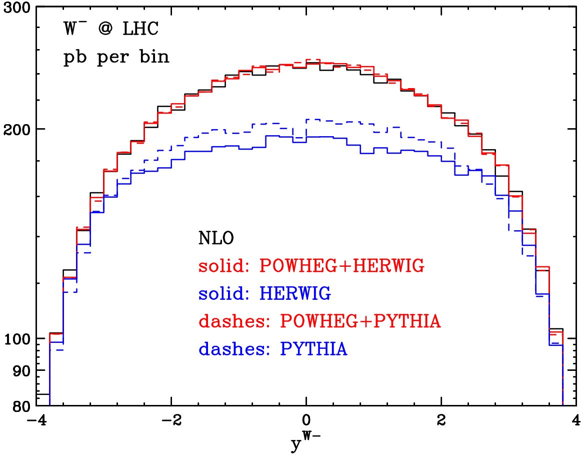
W^- @ LHC, POWHEG interfaced to PYTHIA



- PYTHIA includes hard ME corrections in a POWHEG-like fashion. Nevertheless wrong normalization !
- Mismatch at low p_T: could be due to the fact that POWHEG Sudakov has Next-to-Leading-Log accuracy [Catani et al., Nucl.Phys.B349 (1991)]

RESULTS: POWHEG VS. NLO VS. SMC'S

W^- @ LHC, no K-factors



- No need of K-factors to reach NLO accuracy in inclusive observables

Conclusions (and future prospects)

- POWHEG is a valid method to include NLO corrections in SMC's
- It is independent from the Parton Shower used
- It outputs only positive weighted events, just as traditional (LO) SMC's

  $\rightarrow$ in principle it works not only for QCD corrections

- Single-top and $Z + 1$ jet hadroproduction are close to be completed
- To implement $Z + 1$ jet, we have set up a general framework that should give the possibility to implement an arbitrary NLO calculation without being an expert of the method itself.
- Our codes can be found at

  `http://moby.mib.infn.it/~nason/POWHEG`

END

A TECHNICALITY

PROOF OF POWHEG NLO ACCURACY

$$\langle \mathcal{O} \rangle_{POW} = \int d\Phi_n \, \bar{B}(\Phi_n) \left\{ \Delta(\Phi_n; p_T^{min}) \mathcal{O}(\Phi_n) + \int_{p_T^{min}} \Delta(\Phi_n; k_T) \frac{R(\Phi_{n+1})}{B(\Phi_n)} \mathcal{O}(\Phi_{n+1}) d\Phi_r \right\}$$

$$= \int d\Phi_n \, \bar{B}(\Phi_n) \left\{ \left[\Delta(\Phi_n, p_T^{min}) + \int_{p_T^{min}} \Delta(\Phi_n; k_T) \frac{R(\Phi_{n+1})}{B(\Phi_n)} \, d\Phi_r \right] \mathcal{O}(\Phi_n) \right.$$

$$\left. \int_{p_T^{min}} \Delta(\Phi_n; k_T) \frac{R(\Phi_{n+1})}{B(\Phi_n)} \left[\mathcal{O}(\Phi_{n+1}) - \mathcal{O}(\Phi_n) \right] \, d\Phi_r \right\}$$

but

$$\int_{p_T^{min}} d\Phi_r \frac{R(\Phi_{n+1})}{B(\Phi_n)} \Delta(\Phi_n; k_T) = \mathrm{P}_{emis}(k_T > p_T^{min}) = 1 - \Delta(\Phi_n; p_T^{min})$$

$$\frac{\bar{B}(\Phi_n)}{B(\Phi_n)} = 1 + \mathcal{O}(\alpha_s)$$

$\Delta \approx 1$ far from singular region and $\mathcal{O}(\Phi_{n+1}) - \mathcal{O}(\Phi_n) \to 0$ when $k_T \to 0$

so

$$\Big[\langle \mathcal{O} \rangle_{POW} \Big]_{\mathcal{O}(\alpha_s)} = \int d\Phi_n \left\{ B(\Phi_n) + V(\Phi_n) + \int \left[R(\Phi_{n+1}) - C(\Phi_{n+1}) \right] d\Phi_r \right\} \mathcal{O}(\Phi_n)$$

$$+ \int d\Phi_n d\Phi_r R(\Phi_{n+1}) \left[\mathcal{O}(\Phi_{n+1}) - \mathcal{O}(\Phi_n) \right] \stackrel{!}{=} \langle \mathcal{O} \rangle_{NLO}$$

MC@NLO - POWHEG

MC@NLO

Take the NLO formula and “add and subtract” the shower first emission contribution

1. Generate the first emission according to

$$d\sigma_{MC@NLO} = d\Phi_n \left\{ B(\Phi_n) + V(\Phi_n) + \int d\Phi_r \left[R_{MC}(\Phi_n, \Phi_r) - C(\Phi_n, \Phi_r) \right] \right\}$$
$$+ \quad d\Phi_n \, d\Phi_r \left\{ R(\Phi_n, \Phi_r) - R_{MC}(\Phi_n, \Phi_r) \right\}$$

   $\rightarrow$ double-counting is avoided ($R - R_{MC}$).

2. Apply the shower algorithm, starting with t equal to the hardness of the generated phase space point
   $\rightarrow$ (medium-)high-p_T jets will come not only from real-like events

It works! Several processes implemented, no conceptual problems.
Nevertheless, there are 2 “undesired” features:

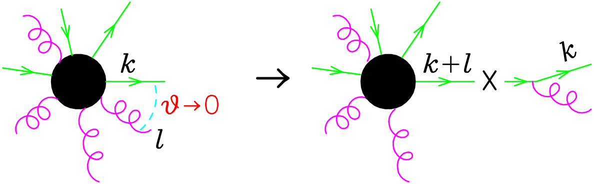
- the difference $R - R_{MC}$ can be negative $\rightarrow$ negative weighted events
- to perform the subtraction, use the kinematics inherited from the SMC $\rightarrow$ specific Parton Shower dependence (HERWIG)

POWHEG

- We interface POWHEG to the shower using the Les Houches interface (SCALUP is the variable that controls subsequent vetoes)
- when interfaced to angular-ordered Shower, need of a p_T-vetoed PS + truncated Shower to generate correctly soft radiation at large angle; implemented in HERWIG++

SHOWER BASICS: COLLINEAR FACTORIZATION (1/2)

QCD emissions are enhanced near the collinear limit and cross sections factorize near this limit:



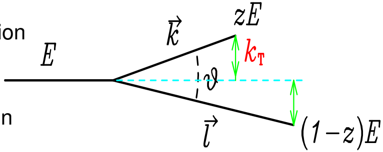
$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \to |\mathcal{M}_n|^2 d\Phi_n \frac{\alpha_s}{2\pi} \frac{dt}{t} P_{q,qg}(z) dz \frac{d\varphi}{2\pi}$$

where

$z = k^0/(k^0 + l^0)$ quark energy fraction

$t = \{(k+l)^2, l_T^2, E^2\theta^2\}$ splitting hardness

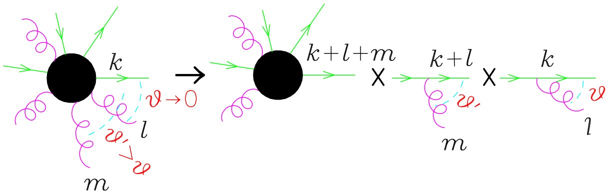
$P_{q,qg}(z) = C_{\rm F}\dfrac{1+z^2}{1-z}$ AP splitting function



$t \to 0$: collinear limit, $z \to 1$: soft limit (ignore for a moment)

Shower basics: collinear factorization (2/2)

If another collinear gluon is emitted off the quark leg with a smaller angle ($\theta, \theta' \to 0$, $\theta' > \theta$), we can iterate the previous formula:



$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \to |\mathcal{M}_{n-1}|^2 d\Phi_{n-1} \left(\frac{\alpha_s}{2\pi} \frac{dt'}{t'} P_{q,qg}(z') dz' \frac{d\varphi'}{2\pi} \right) \left(\frac{\alpha_s}{2\pi} \frac{dt}{t} P_{q,qg}(z) dz \frac{d\varphi}{2\pi} \right) \Theta(t' -$$

$\Rightarrow$ Collinear emissions can be described by a factorized integral ordered in t

Within this approximation, the cross section for a hard process dressed with n collinear emissions goes as

$$\sigma_n \approx \sigma_0 \, \alpha_s^n \int_{t_0}^{Q^2} \frac{dt_1}{t_1} \int_{t_0}^{t_1} \frac{dt_2}{t_2} ... \int_{t_0}^{t_{n-1}} \frac{dt_n}{t_n} = \sigma_0 \, \alpha_s^n \frac{1}{n!} \left(\log \frac{Q^2}{t_0} \right)^n$$

where

- $Q^2 =$ hardness of the hard scattering (and upper cutoff for the ordering variable t)
- $t_0 \approx \Lambda^2_{QCD}$ is an infrared cutoff
- σ_n / σ_0 is of order 1; $(\alpha_s \log)^n$ is called leading-log approximation (LLA)

Shower basics: Virtual and Soft corrections

Virtual Corrections

It can be shown that the inclusion of virtual corrections, in the LLA, can be obtained by:

- At each vertex, calculate the splitting probability with

  $$\alpha_s(Q^2) \to \alpha_s(t)$$

  where t is the hardness of the incoming line;
- For each intermediate line, include the Sudakov form factor

  $$\Delta_a(t_i, t_{i+1}) = \exp\left[-\sum_{(bc)} \int_{t_{i+1}}^{t_i} \frac{dt'}{t'} \int \frac{\alpha_s(t')}{2\pi} P_{a,bc}(z)\, dz\right]$$

  where t_i and t_{i+1} are the virtualities of the vertexes where the line respectively begins and ends

Soft emissions

- Mueller (1981) showed that angular ordering is the correct choice ($t = \theta$):

  $$d\mathsf{P}_{emis} = \frac{d\theta}{\theta} \frac{\alpha_s(p_T^2)}{2\pi} P(z) dz \ , \quad \theta_1 > \theta_2 > \theta_3 \ldots \ , \quad p_T^2 = E^2 z^2 (1-z)^2 \theta^2$$

  The argument of α_s is chosen equal to p_T^2 for a correct treatment of the charge renormalization in the soft region.

PROBABILISTIC INTERPRETATION OF THE SUDAKOV FACTOR

There is a simple argument to understand why the Sudakov factor contains LL virtual corrections:

- Probability of one emission off a quark line, in the interval δt, at order α_s, in the LLA, integrated over z and φ:

$$d\mathsf{P}_{emis}(t+\delta t, t) = \frac{\alpha_s(t)}{2\pi}\frac{\delta t}{t}\int P_{q,qg}(z)dz$$

- Probability of no emission in δt:

$$d\mathsf{P}_{no\ emis}(t+\delta t, t) = 1 - \frac{\alpha_s(t)}{2\pi}\frac{\delta t}{t}\int P_{q,qg}(z)dz$$

  Virtual corrections are included here because there is a power of α_s but no splitting.

- The probability of no emission between two values t_1 and t_2 of the ordering scale is given by

$$\mathsf{P}_{no\ emis}(t_1, t_2) = \lim_{N\to\infty}\prod_{i=1}^{N}\left[1 - \frac{\delta t}{t_i}\frac{\alpha_s(t_i)}{2\pi}\int P_{q,qg}(z)dz\right]$$

  where we have divided the finite interval $[t_2, t_1]$ in N small intervals $\delta t = (t_1 - t_2)/N$ and where t_i is a point in the i-th intervall.

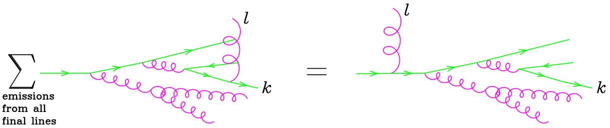
- Taking the limit $N \to \infty$ leads to

$$\mathsf{P}_{no\ emis}(t_1, t_2) = \exp\left[-\int_{t_{i+1}}^{t_i}\frac{dt'}{t'}\int\frac{\alpha_s(t')}{2\pi}P_{q,qg}(z)\,dz\right] \equiv \Delta_q(t_1, t_2) \in [0, 1]$$

$\Rightarrow$ the Sudakov factor $\Delta(t_1, t_2)$ is the probability of non emitting between t_1 and t_2.

COLOR COHERENCE

Up to now, all the approximations we did allowed to treat branchings incoherently.
Soft emissions from final-state-partons add coherently:



In the above figure, the soft large-angle gluon sees the net colour charge of the initial quark, and not the charges of each emitter.

- In non angular-ordered Shower, this is not taken into account $\rightarrow$ need of corrections to the algorithm without spoiling the collinear accuracy.
- If the Shower is angular-ordered, the coherence is built-in: large-angle soft emissions are generated first.
- The hardest emission (highest p_T), in general, happens later.

Among many, there are two commonly used Standard Monte Carlo (SMC) event generators: `HERWIG` (Marchesini, Webber 1988) and `PYTHIA` (Bengtsson, Sjostrand 1987). They differ in the choice of the ordering variable and in the hadronization model, but the main Parton Shower algorithm is the same:

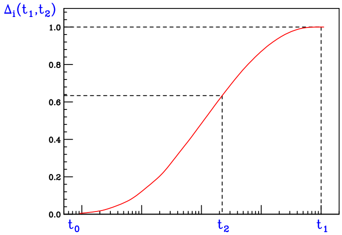
↓

The Shower algorithm

We know how to deal between two emission at different scales: in that interval of hardness, the 'exact' probability of having no splittings is given by the Sudakov form factor. The final rules are:

- generate a hard event according to $d\sigma_B = |\mathcal{M}_B|^2 d\Phi_B$.
  This automatically fixes the hard scale Q^2 for the current event.
- for each colored parton i, generate a shower:
  Key observation: $\mathrm{P}_{emis}(t'|t)dt' \equiv \mathrm{P}_{no\ emis}(t,t')d\mathrm{P}_{emis}(t') \stackrel{!}{=} d\Delta(t,t')$

1. set $t = Q^2$
2. extract a uniform random number in $[0,1]$
3. solve the equation $\Delta_i(t,t') = r$ for t'
4. if $t' < t_0$, don't split (we are at the hadronization scale)
5. if $t' > t_0$, generate z and (jk) with probability $P_{i,jk}(z)$ and φ flat in $[0, 2\pi]$
6. restart a shower from each new branch, setting the new ordering parameter $t = t'$
7. when all legs have $t \simeq t_0$, apply the hadronization model



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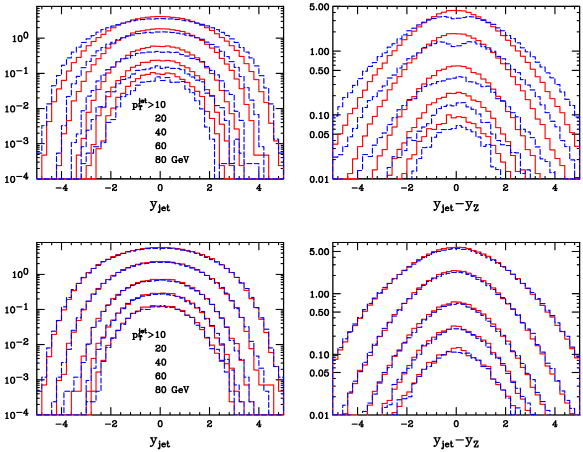
Sudakov suppression of low k_T radiation:

$$\mathrm{P}_{emis}(k_T^2 \stackrel{\sim}{<} t_0) = \Delta(Q^2, t_0) \to 0$$

RAPIDITY & RAPIDITY DIFFERENCE @ TEV (Z)



POWHEG + HERWIG

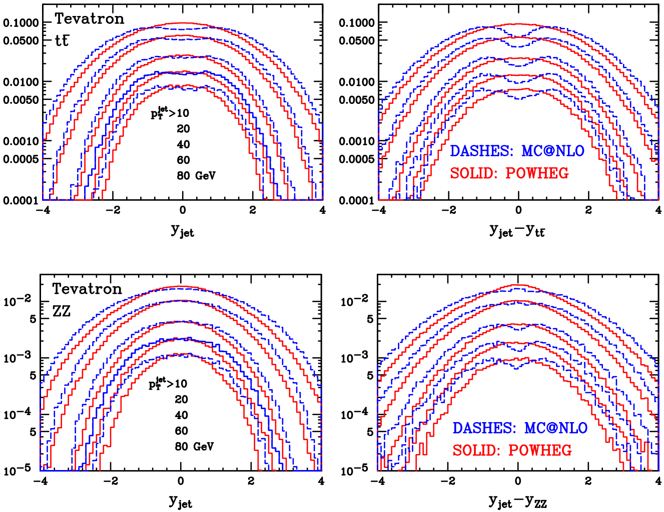
MC@NLO

POWHEG + PYTHIA

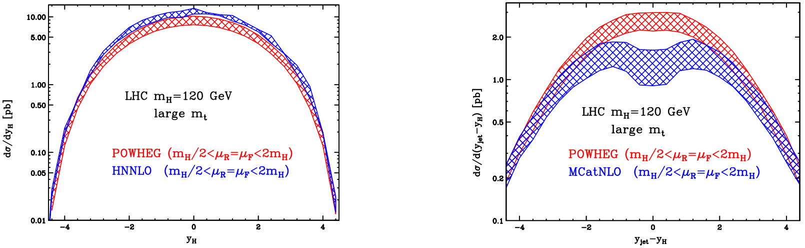
PYTHIA $\times$ 1.3

- MC@NLO distribution are flatter in y_{jet} and have a dip in $y_{jet} - y_Z$. It seems a general feature of MC@NLO, already noticed...

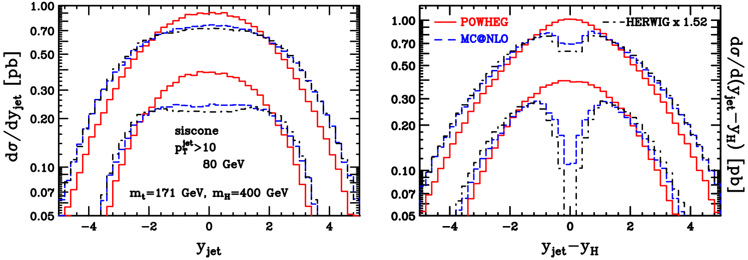
Rapidity & rapidity difference @ TeV ($Q\bar{Q}$, ZZ)



HARDEST JET RAPIDITY DIP

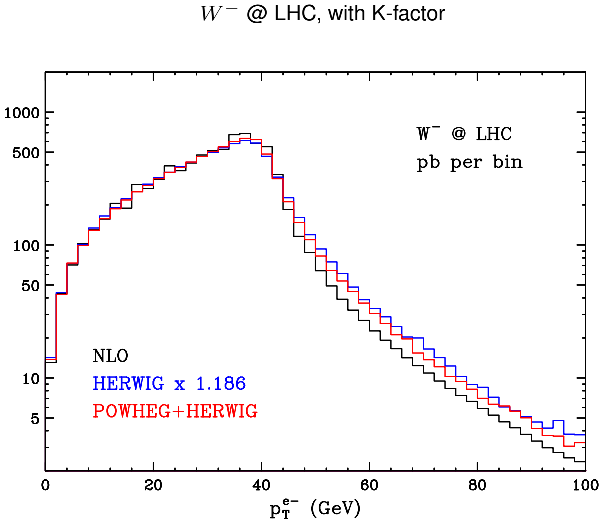


NNLO results obtained using `HNNLO` [Catani and Grazzini, arXiv:0802.1410]

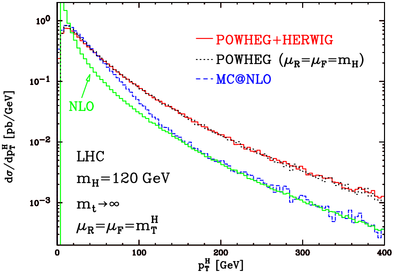


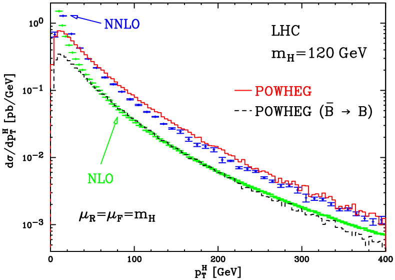
- It seems a feature of HERWIG; MC@NLO only partially fill the dip

RESULTS: POWHEG VS. NLO VS. SMC'S



HIGGS BOSON HIGH$-p_T$ MISMATCH





$$\bar{B}(\Phi_n) = B(\Phi_n) + V(\Phi_n) + \int \left[R(\Phi_n, \Phi_r) - C(\Phi_n, \Phi_r)\right] \, d\Phi_r$$

$$d\sigma = \bar{B}(\Phi_n) \, d\Phi_n \left\{\Delta(\Phi_n; p_T^{min}) + \Delta(\Phi_n; k_T) \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} \, d\Phi_r\right\}$$

if $p_T \gg 1 \quad \Rightarrow \quad \Delta(\Phi_n; k_T) \approx 1$ and

$$d\sigma_{\rm rad} \approx \frac{\bar{B}(\Phi_n)}{B(\Phi_n)} R(\Phi_n, \Phi_r) \, d\Phi_n \, d\Phi_r \approx \{1 + \mathcal{O}(\alpha_s)\} \, R(\Phi_n, \Phi_r) \, d\Phi_n \, d\Phi_r$$

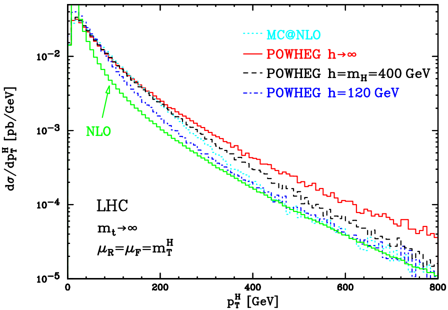
Fortunately this mismatch brings the POWHEG curve close to the NNLO result

Reduction of real contribution in the Sudakov FF

We try to see if we were able to reproduce the NLO result.

$$R = [R \times F] + [R \times (1-F)]$$
$$= R_F + R_{\text{reg}}$$

$$F < 1,$$
$$F \to 1 \text{ when } k_T \to 0,$$
$$F \to 0 \text{ when } k_T \to \infty$$
$$F = \frac{h^2}{k_T^2 + h^2}$$



$$d\sigma_{POW} = d\sigma_{\bar{B}} + d\sigma_{\text{reg}}$$

$$d\sigma_{\bar{B}} = d\Phi_n \; \bar{B}_{R_F} \left\{ \Delta_{R_F}(\Phi_n; p_T^{min}) + \Delta_{R_F}(\Phi_n; k_T) \frac{R_F(\Phi_n, \Phi_r)}{B(\Phi_n)} \, d\Phi_r \right\} \qquad \leftarrow \text{low } k_T$$

$$d\sigma_{\text{reg}} = R_{\text{reg}}(\Phi_n, \Phi_r) \, d\Phi_n \, d\Phi_r \qquad \leftarrow \text{high } k_T$$

POWHEG seems a flexible method → Good news in view of more complicated processes