

Higher order QCD corrections to the Drell-Yan process

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Outline

- Introduction
- NLL+LO resummation
- NNLO calculation
- Summary & Outlook

Introduction

The production of vector bosons in hadron collision is important for physics studies within and beyond the SM

- Large production rates and clean experimental signatures make these processes standard candles for detector calibration
- Possible use as luminosity monitor

Among the various distribution an important role is played by the transverse momentum spectrum

→ Uncertainties on the W spectrum directly affect W mass

When considering the transverse momentum spectrum of the vector boson it is important to distinguish two region of transverse momenta

The region $q_T \sim m_V$

To have $q_T \neq 0$ the vector boson has to recoil against at least one parton \rightarrow the LO is $\mathcal{O}(\alpha_S)$

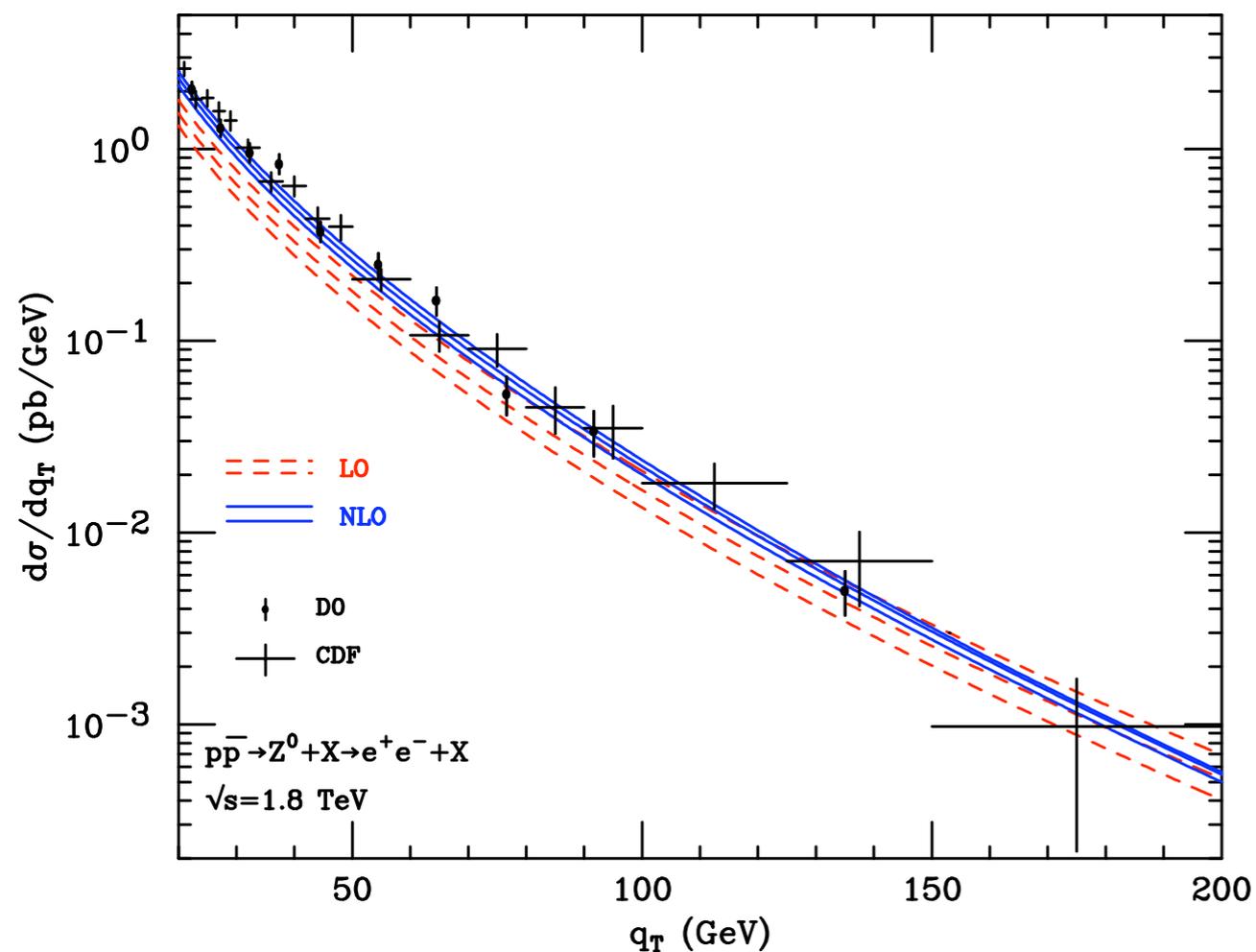
NLO corrections have been known for many years

K.Ellis, G.Martinelli, R.Petronzio (1983)

P.B.Arnold, M.H. Reno (1989)

R.J.Gonsalves, J.Pawlowski, C.F.Wai (1989)

Implemented in the general purpose MCFM program



Tevatron Z data nicely described down to $q_T \sim 20$ GeV

G.Bozzi, S.Catani, G.Ferrera, D. de Florian, MG

scale uncertainty computed by varying μ_R and μ_F in the range $0.5 \leq \mu_F, \mu_R \leq 2$ with the constraint $0.5 \leq \mu_F/\mu_R \leq 2$

The region $q_T \ll m_V$

The small q_T region is the most important because it is here that the bulk of the events is expected

When $q_T \ll m_V$ large logarithmic corrections of the form $\alpha_S^n \ln^{2n} m_V^2 / q_T^2$ appear that originate from soft and collinear emission

→ the perturbative expansion becomes not reliable

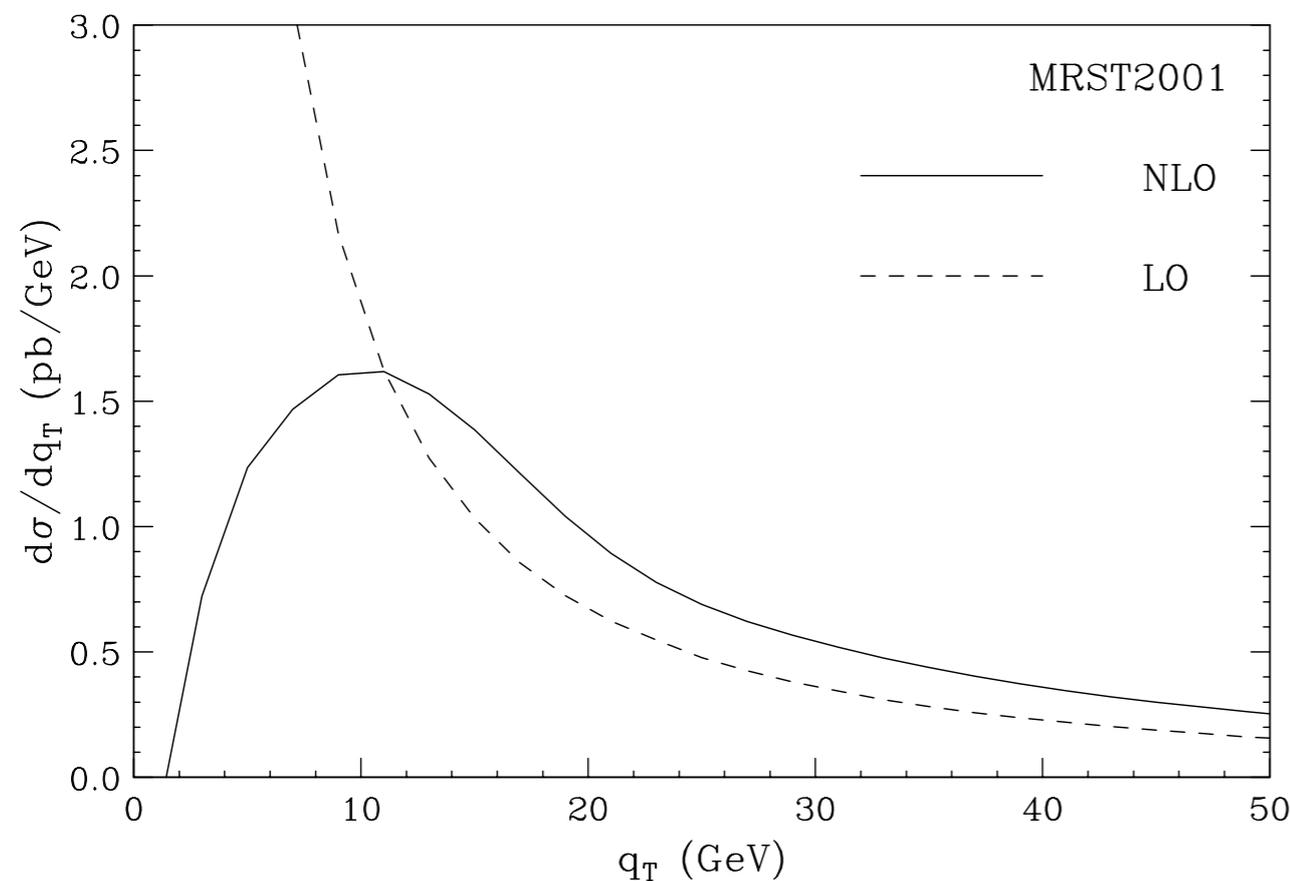
This is a general problem in the production of systems of high mass Q^2 in hadronic collisions → **RESUMMATION IS NEEDED**

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$$\text{LO: } \frac{d\sigma}{dq_T} \rightarrow +\infty \text{ as } q_T \rightarrow 0$$

$$\text{NLO: } \frac{d\sigma}{dq_T} \rightarrow -\infty \text{ as } q_T \rightarrow 0$$

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The resummation formalism has been developed in the eighties

Y.Dokshitzer, D.Diakonov, S.I.Troian (1978)

G. Parisi, R. Petronzio (1979)

G. Curci, M.Greco, Y.Srivastava(1979)

J. Collins, D.E. Soper, G. Sterman (1985)

As it is customary in QCD resummations one has to work in a conjugate space in order to allow the kinematics of multiple gluon emission to factorize

In this case, to exactly implement momentum conservation, the resummation has to be performed in impact parameter b -space

The standard (CSS) formalism has several disadvantages:

- The resummation coefficients are process dependent D. de Florian, MG (2000)
- The integral over b involves and extrapolation of the parton distributions to the non-perturbative region
- The resummation effects are large also at small b
 - ➔ - No control on the normalization
 - Problems in the matching to the PT result

Our formalism

A version of the b-space formalism has been proposed that overcomes these problems

Parton distributions factorized at $\mu_F \sim m_V$

S.Catani, D. de Florian, MG (2000)
G. Bozzi, S.Catani, D. de Florian, MG(2005)

$$\frac{d\hat{\sigma}_{ac}^{(\text{res.})}}{dq_T^2} = \frac{1}{2} \int_0^\infty db b J_0(bq_T) \mathcal{W}_{ac}(b, M, \hat{s}; \alpha_S(\mu_R^2), \mu_R^2, \mu_F^2)$$

lepton pair invariant mass 

$$\mathcal{W}_N^V(b, M; \alpha_S(\mu_R^2), \mu_R^2, \mu_F^2) = \mathcal{H}_N^V(M, \alpha_S(\mu_R^2); M^2/\mu_R^2, M^2/\mu_F^2, M^2/Q^2)$$

process dependent 

$$\times \exp\{\mathcal{G}_N(\alpha_S(\mu_R^2), L; M^2/\mu_R^2, M^2/Q^2)\}$$

where the large logs
are organized as:

$$\mathcal{G}_N(\alpha_S, L; M^2/\mu_R^2, M^2/Q^2) = L g^{(1)}(\alpha_S L)$$

universal 

$$+ g_N^{(2)}(\alpha_S L; M^2/\mu_R^2, M^2/Q^2) + \alpha_S g_N^{(3)}(\alpha_S L; M^2/\mu_R^2, M^2/Q^2) + \dots$$

with $L = \ln M^2 b^2 / b_0^2$  $\tilde{L} = \ln(1 + Q^2 b^2 / b_0^2)$ and $\alpha_S = \alpha_S(\mu_R)$

resummation scale 

- The form factor takes the same form as in threshold resummation
-  - Unitarity constraint enforces correct total cross section
- Allows a consistent study of perturbative uncertainties

The resummed and fixed order calculations can then be combined to achieve uniform theoretical accuracy over the entire range of q_T

$$\frac{d\hat{\sigma}}{dq_T^2} = \frac{d\hat{\sigma}^{(\text{res.})}}{dq_T^2} + \frac{d\hat{\sigma}^{(\text{fin.})}}{dq_T^2}$$

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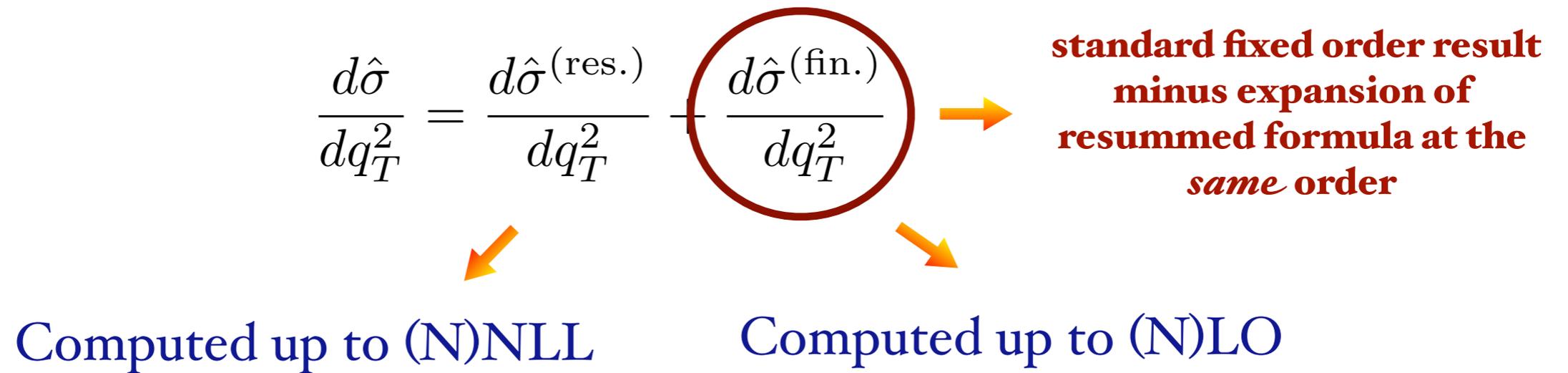
$$\frac{d\hat{\sigma}}{dq_T^2} = \frac{d\hat{\sigma}^{(\text{res.})}}{dq_T^2} - \frac{d\hat{\sigma}^{(\text{fin.})}}{dq_T^2} \rightarrow \text{standard fixed order result} \\ \text{minus expansion of resummed formula at the} \\ \text{same order}$$

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Computed up to (N)NLL **Computed up to (N)LO**

**standard fixed order result
minus expansion of
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Computed up to (N)NLL Computed up to (N)LO

We have applied this formalism up to NNLL+NLO for Higgs production

G.Bozzi, S.Catani, D. de Florian, MG (2000)

Our goal is to do the same for vector boson production, since all the perturbative information is available

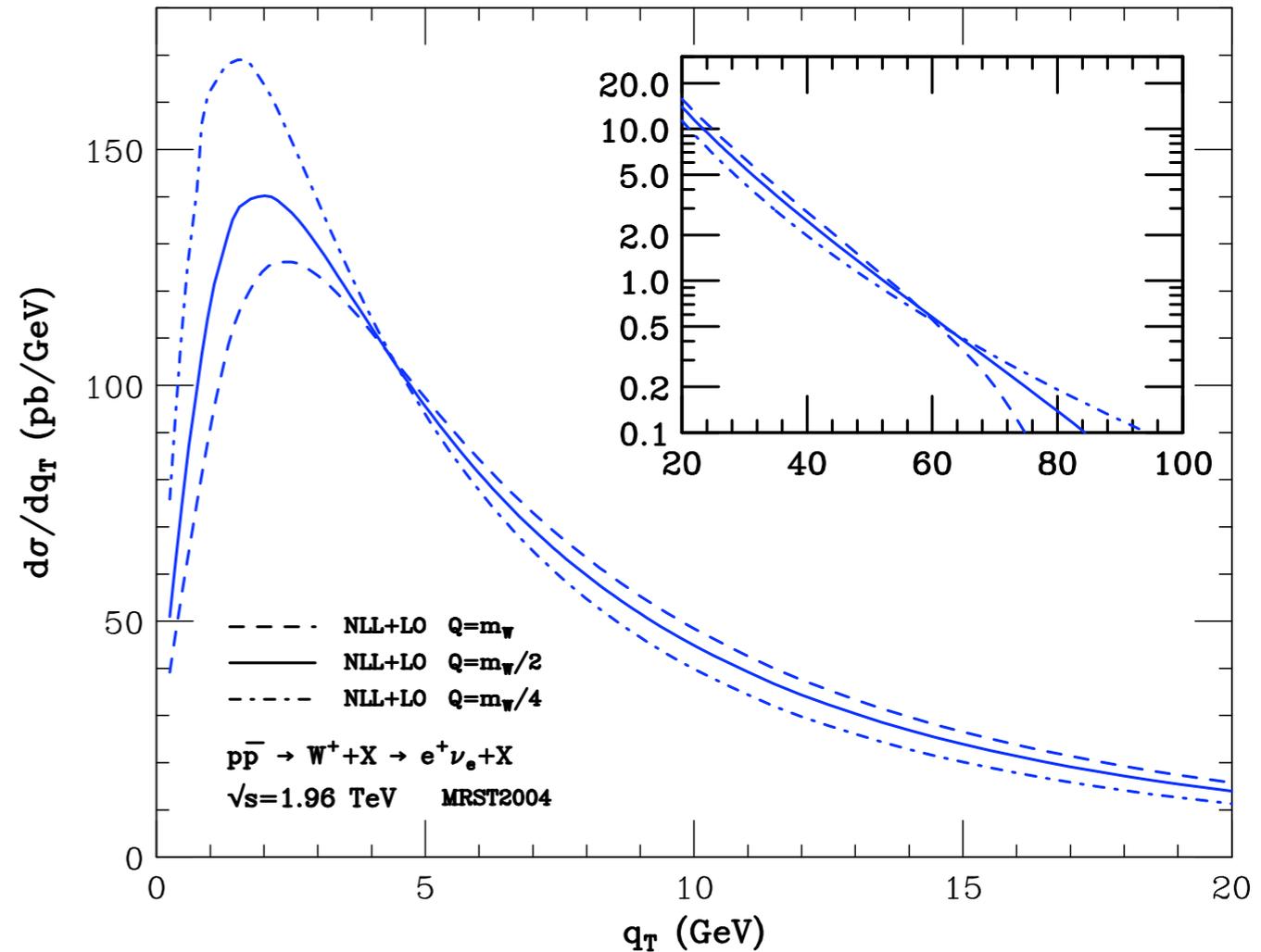
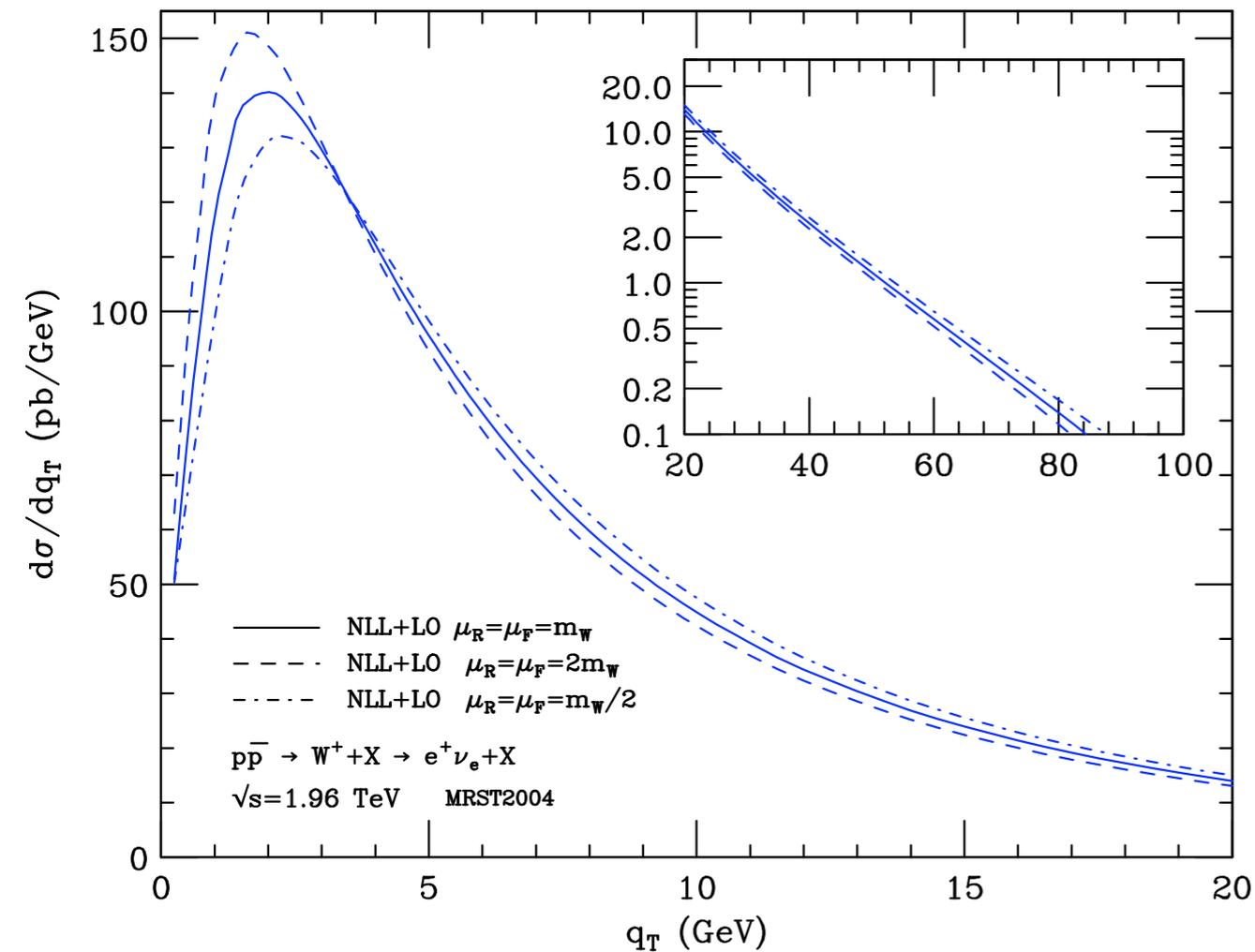
The only missing ingredient was the second order hard coefficient $\mathcal{H}^{V(2)}$

→ recently computed !

S.Catani, L.Cieri, G.Ferrera,
D. de Florian, MG (2009)

For the moment we only have results up to NLL+LO but we are now ready to go to NNLL+NLO

NLL+LO results for W production



The perturbative uncertainty is estimated by varying μ_F and μ_R

The same is done for the resummation scale Q → allows to estimate effect of missing higher order logarithmic contributions

Uncertainty from μ_F and μ_R variations is about $\pm 5\%$

Uncertainty from Q variations is larger and about $\pm 15\%$

Note that this is basically the same formal accuracy of MC@NLO and POWEG

NEW:

A fully exclusive NNLO calculation

S.Catani, L.Cieri, G.Ferrera,
D. de Florian, MG (2009)

NNLO corrections for vector boson production are known and implemented in FEWZ

K.Melnikov, F.Petriello (2006)

We have recently completed an independent NNLO calculation by using a recently proposed version of the subtraction method

S.Catani, MG (2007)

Our calculation includes $\gamma - Z$ interference, finite width effects, the leptonic decay of the vector boson and the corresponding spin correlations

The calculation is implemented in a parton level event generator with which we can apply arbitrary cuts on the final state leptons and the associated jet activity and plot the required distributions as bin histograms



To be released soon !

The method

We use an extension of the subtraction formalism recently proposed and applied for Higgs boson production

S. Catani, MG (2007), MG (2008)

Start by observing that when $q_T \neq 0 \rightarrow d\sigma_{(N)NLO}^V|_{q_T \neq 0} = d\sigma_{(N)LO}^{V+jets}$

IR singularities of NLO type already handled and cancelled in $d\sigma_{(N)LO}^{V+jets}$

 Additional singularities at $q_T \rightarrow 0$ can be handled through a counterterm constructed from resummation coefficients

Same hard coefficient appearing in resummation formula

$$d\sigma_{(N)NLO}^V = \mathcal{H}_{(N)NLO}^V \otimes d\sigma_{LO}^V + \left[d\sigma_{(N)LO}^{V+jets} - d\sigma_{(N)LO}^{CT} \right]$$


**Born like contribution
depending on the hard function \mathcal{H}^V**

**Finite as $q_T \rightarrow 0$: it is the most
difficult to evaluate numerically**

Results: W production at the Tevatron

Define $m_T = \sqrt{2p_T^l p_T^{\text{miss}}(1 - \cos \phi)}$

Cuts: $p_T^{\text{miss}} > 25 \text{ GeV}$

$p_T^l > 20 \text{ GeV}$ $|\eta| < 2$

Note that at LO $\phi = \pi$

 $p_T^{\text{miss}} > 25 \text{ GeV}$

gives $m_T > 50 \text{ GeV}$

In the presence of such a kinematical boundary there are perturbative instabilities at NLO and NNLO

Below the boundary the NNLO effects is large +40% at $m_T \sim 30 \text{ GeV}$



This is because in this region the calculation is NLO !

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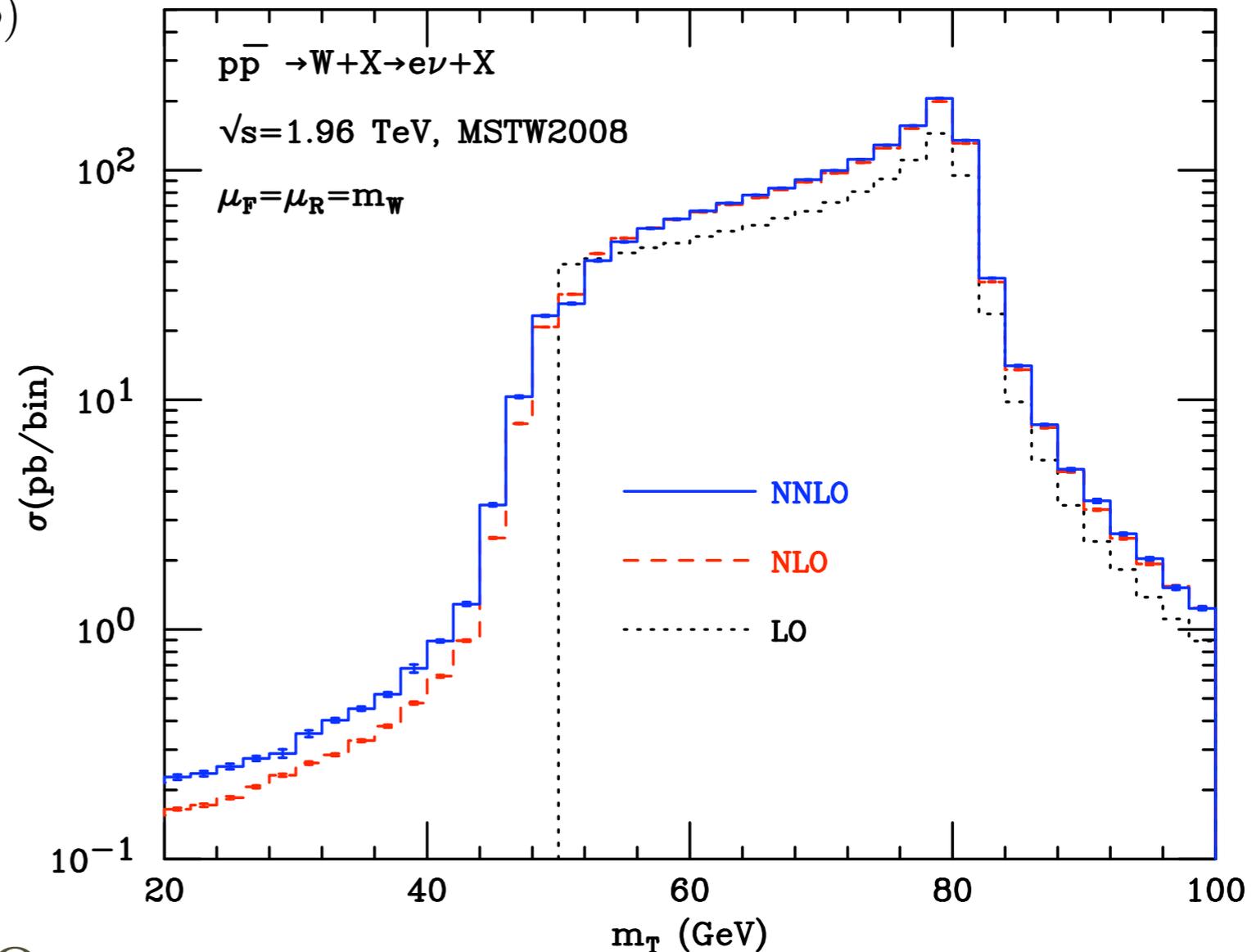
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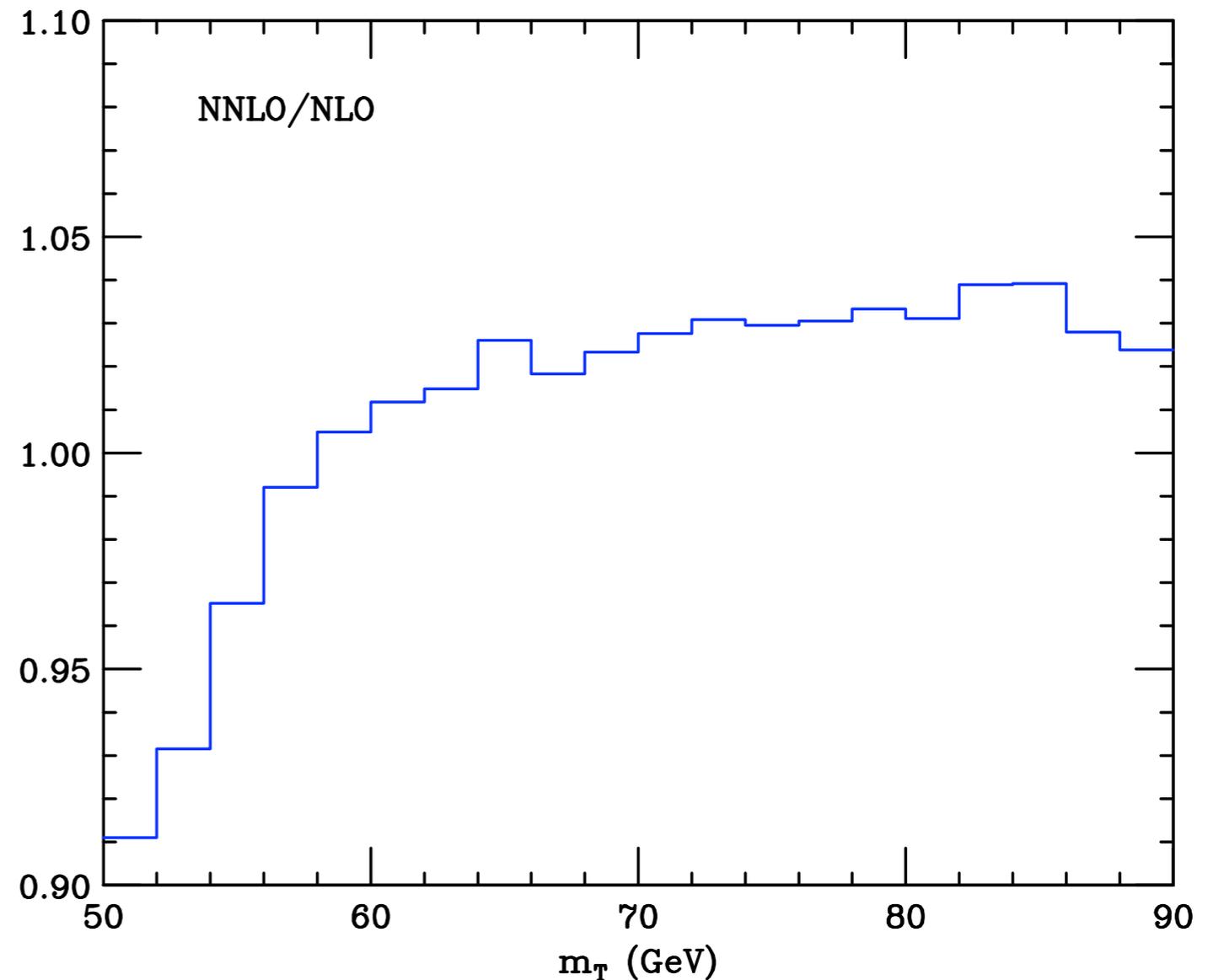
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Summary & Outlook

Our work on the DY process is (slowly) going on

- Transverse momentum resummation at NLL+LO done
 - Two-loop calculation of hard coefficients completed
- This result allowed us to complete the fully exclusive NNLO calculation: **PUBLIC CODE AVAILABLE SOON**

To be done:

- Extend the resummed calculation to NNLL+NLO
- Include vector boson decay
- Include NP effects and compare to data