Ambiguities in resummation prescriptions

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In collaboration with:

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Resummation formalism

A typical observable (e.g. Drell-Yan cross-section) ($x = \frac{Q^2}{S}$)

$$\sigma(x, Q^2) = \int_x^1 \frac{dz}{z}\, \mathcal{L}(z, Q^2)\, \hat{\sigma}\left(\frac{x}{z}, \alpha_s(Q^2)\right)$$

$\mathcal{L}(z, Q^2)$ is a luminosity (convolution of pdfs) and $\hat{\sigma}\left(z, \alpha_s(Q^2)\right)$ is the partonic cross-section.

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These large logs need to be resummed

Resummation is usually preformed in Mellin space in order to have factorization:

$$\hat{\sigma}^{\text{res}}\left(N, \alpha_s(Q^2)\right) = \exp \mathcal{S}(N, Q^2)$$

$\mathcal{S}(N, Q^2)$: Sudakov exponent

Landau pole

$$\mathcal{S}(N,Q^2) = \int_1^{N^2} \frac{dn}{n}\, g\left(\alpha_s\left(\frac{Q^2}{n}\right)\right)\ , \qquad g(\alpha_s)\ \text{analytic in}\ \alpha_s$$

Landau pole singularity $\Rightarrow$ branch cut in N-space.

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For example the quantity $\gamma(N,\alpha_s(Q^2))=\frac{\partial\mathcal{S}(N,Q^2)}{\partial\log Q^2}$ at leading log (LL) approximation

$$\gamma_{\rm LL}(N,\alpha_s(Q^2))=A\;\log\left(1+\beta_0\alpha_s(Q^2)\log\frac{1}{N}\right)$$

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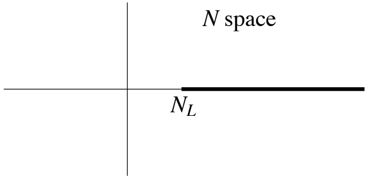
$$N > N_L \equiv \exp \frac{1}{\beta_0 \alpha_s(Q^2)}$$



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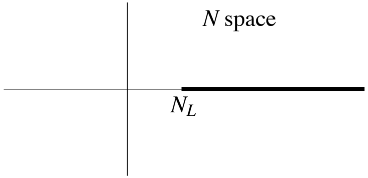

The Mellin inverse does NOT exist

Connection with divergence of perturbative expansion

We can expand in series of $\alpha_s(Q^2)$ and invert term by term:

$$\mathcal{M}^{-1}\left[\gamma_{\rm LL}\right] = -A\sum_{k=1}^{\infty}\frac{(-\beta_0\alpha_s(Q^2))^k}{k}\,\mathcal{M}^{-1}\left[\log^k\frac{1}{N}\right]$$

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Approximate the Mellin inversion of the single log at LL:

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and take the sum:

$$\frac{\mathcal{M}^{-1}\left[\gamma_{\rm LL}\right]}{A} = \left[\frac{1}{1-z}\, \frac{\beta_0 \alpha_s(Q^2)}{1+\beta_0\alpha_s(Q^2)\log(1-z)}\right]_+ = \left[\frac{\alpha_s\left(Q^2(1-z)\right)}{1-z}\right]_+$$

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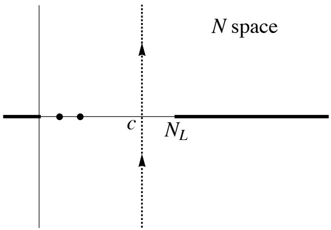
Landau pole!

Minimal prescription

Proposed by S.Catani, M.Mangano, P.Nason, L.Trentadue:

$$\sigma^{\rm MP}(x,Q^2) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN\; x^{-N}\,\mathcal{L}(N,Q^2)\,\hat\sigma^{\rm res}\left(N,\alpha_s(Q^2)\right)$$

with $c < N_L$, as in the figure.



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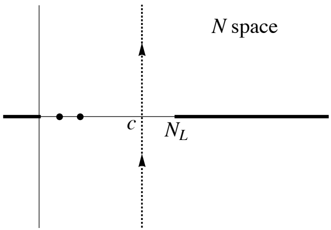

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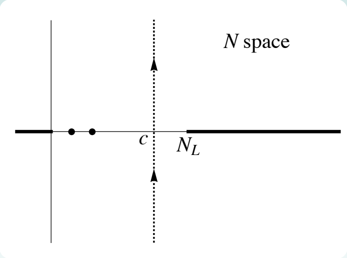

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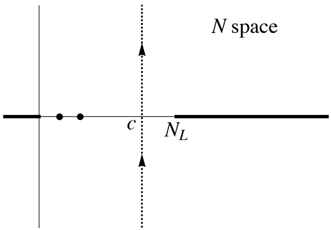

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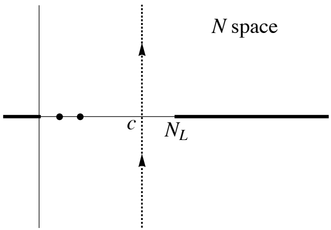

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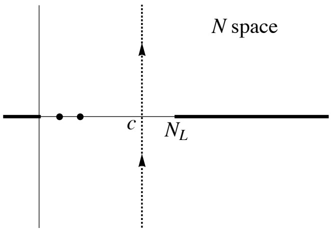

But...

- a non-physical region of the parton cross-section contributes
- problems in numerical implementation

Borel prescription (1)

Generic resummed quantity (for example $\Sigma\left(\bar{\alpha}L\right) = \gamma_{\rm LL}(N, \alpha_s(Q^2))$)

$$\Sigma\left(\bar{\alpha}L\right) = \sum_{k=0}^{\infty} h_k \left(\bar{\alpha}L\right)^k , \qquad \begin{cases} \bar{\alpha} \equiv 2\beta_0\, \alpha_s(Q^2) \\ L \equiv \log \frac{1}{N} \end{cases}$$

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- If the original series converges $\Rightarrow \quad f_{\rm B}(z) = f(z)$

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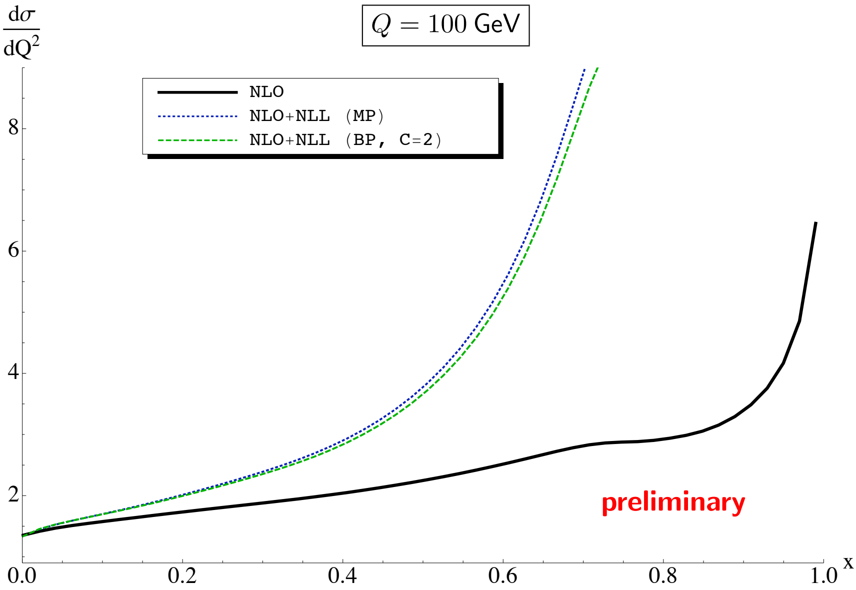
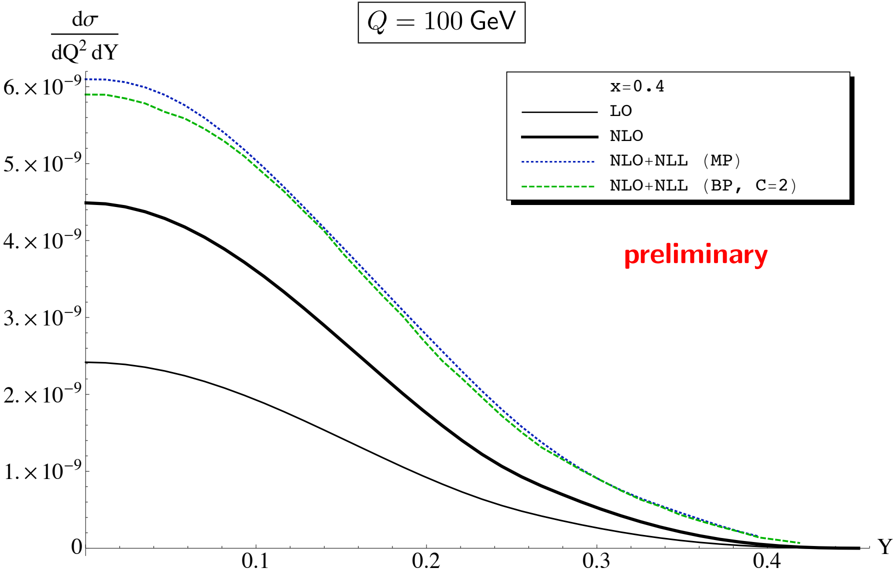
- the cut-off is related to the inclusion of higher-twist terms

$$\exp\left(-\frac{C}{\bar{\alpha}}\right) \simeq \left(\frac{\Lambda^2}{Q^2}\right)^{C/2}$$

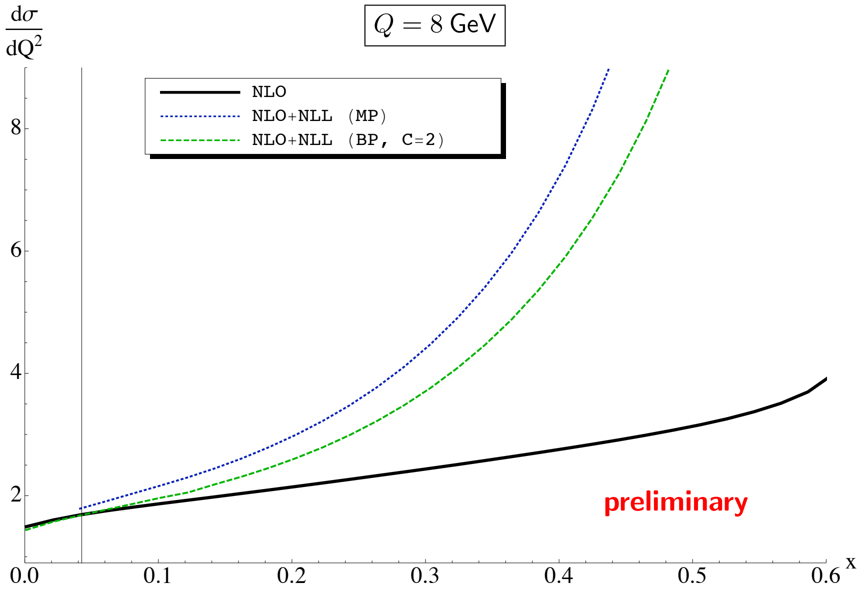
Total cross-section (normalized to LO, cteq6.6 pdfs used)



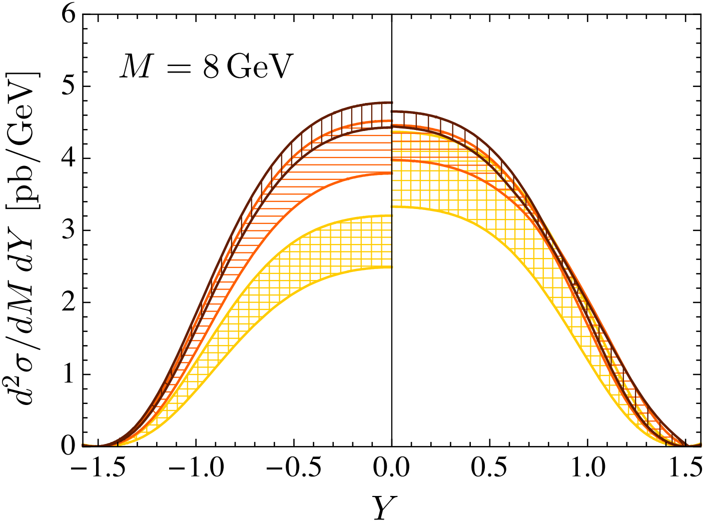
Rapidity distribution (cteq6.6 pdfs used)



Total cross-section (normalized to LO, mrst2001nlo pdfs used)



Rapidity distribution for E866/NuSea (mrst2001nlo pdfs used)



T.Becher, M.Neubert, G.Xu, JHEP **0807** (2008) 030

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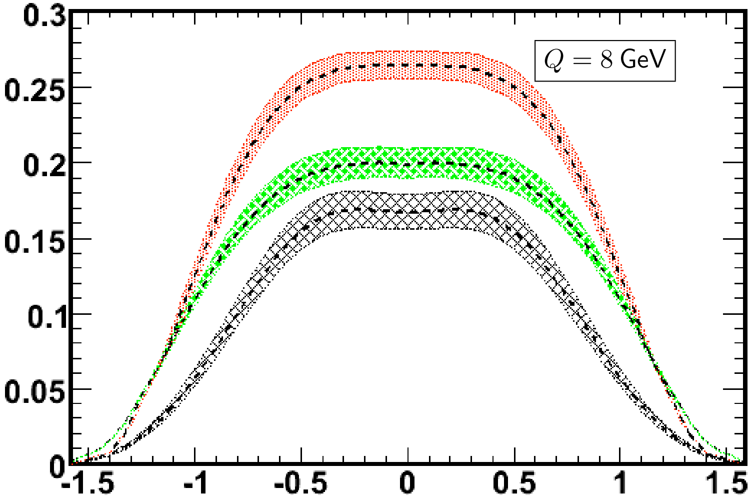

P.Bolzoni, Phys. Lett. B **643** (2006) 325

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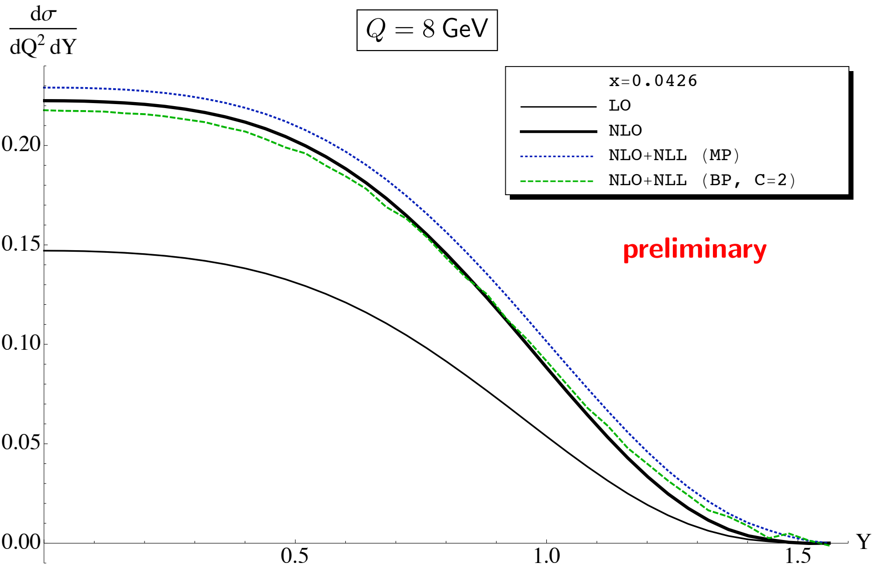

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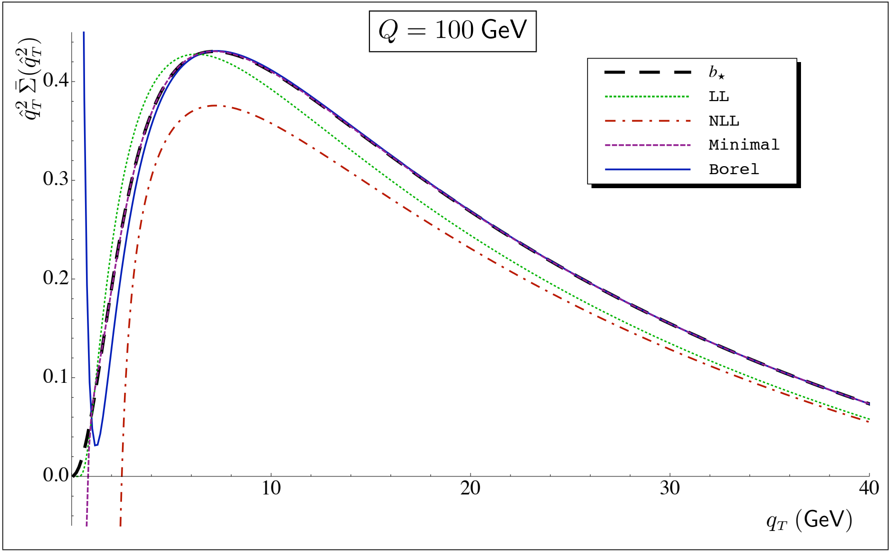

M.Bonvini, S.Forte, G.Ridolfi, Nucl. Phys. B **808** (2009) 347

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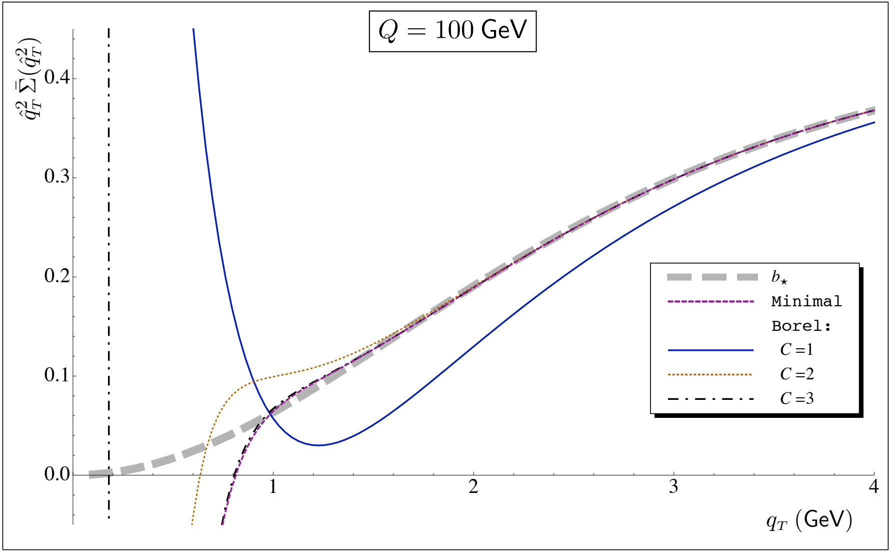

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- Is the ambiguity important?
  - total cross-section (and rapidity distribution): non-negligible
  - transverse momentum distribution: very small

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Main concepts and results

- Perturbative expansions are divergent:
  - ambiguity in the extraction of a finite result
  - some prescriptions to deal with divergence are available
  - estimate ambiguity using different prescriptions
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- Do we need a non-perturbative function?
  - no, for total cross-section and rapidity distribution
  - for transverse momentum distribution only for very small q_T

Spare slides

Minimal prescription: non-physical contribution

$$\begin{aligned}
\sigma^{\rm MP}(x,Q^2) &= \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN\; x^{-N}\,\hat\sigma^{\rm res}\left(N,\alpha_s(Q^2)\right)\;\mathcal{L}(N,Q^2)\\
&= \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN\; x^{-N}\,\hat\sigma^{\rm res}\left(N,\alpha_s(Q^2)\right)\int_0^1 dz\; z^{N-1}\,\mathcal{L}(z,Q^2)\\
&= \int_0^1 \frac{dz}{z}\,\mathcal{L}(z,Q^2)\,\hat\sigma^{\rm res}\left(\frac{x}{z},\alpha_s(Q^2)\right)
\end{aligned}$$

The integral extends from 0 to 1, not from x to 1!

MP vs BP for single logarithm

Using Minimal prescription we get the exact inversion

$$\mathcal{M}^{-1}\left(\log\frac{1}{N}\right)_{\rm MP} = \left[\frac{1}{\log\frac{1}{z}}\right]_+$$

Using Borel prescription we get the more physical result

$$\mathcal{M}^{-1}\left(\log\frac{1}{N}\right)_{\rm BP} = \left[\frac{1}{1-z}\right]_+\left(1-e^{-\frac{C}{\bar\alpha}}\right)$$