Light Pair Corrections to W Mass

work in progress and a few questions

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with O. Nicrosini, F. Piccinini, the HORACE co–authors & two Pavia final year students

Pair correction: theory

Two possible approaches

1. Perturbative calculation ("benchmark", process dependent): up to non–logarithmic (NL) accuracy
2. QED Structure Functions (process independent): Leading Logarithmic (LL) approximation

Ingredients of the perturbative calculation

Consider the leptonic pair correction to the (time-like) high–energy muon vertex in QED, i.e. collision $(\sqrt{s}) \to \mu^+(q_+)\mu^-(q_-)[l^+(p_+)l^-(p_-)]$, where the emitted l^+l^- pairs is undetected (because virtual, or soft/collinear if real)

- real pair correction. If the total energy Δ of the emitted pair is "small", i.e. $\Delta \ll \sqrt{s}$ (soft pairs) then

$$M_{\rm real}^{\rm soft} = M_0 \frac{4\pi\alpha}{k^2} \bar{v}(p_+)\gamma^\mu u(p_-) J_\mu$$

$$J_\mu = \frac{q_{-\mu}}{q_- k - k^2/2} - \frac{q_{+\mu}}{q_+ k - k^2/2} \qquad k = p_+ + p_-$$

- virtual correction: two–loop vertex form factor

$$M_{\rm virtual} = M_0\, 2 \left(\frac{\alpha}{\pi}\right)^2 {\rm Re}\, F(s)$$

Perturbative calculation: correction structure

1. Real correction (after phase space integration of $M_{\rm real}^{\rm soft}$)

$$R = \left(\frac{\alpha}{\pi}\right)^2 \left[c_3 \, (L - 2l)^3 + c_2 (L - 2l)^2 + c_1 (L - 2l) + c_0\right]$$

$$L = \ln(s/m_l^2) \qquad l = \ln(\sqrt{s}/2\Delta)$$

2. Virtual correction

$$V \equiv 2 \left(\frac{\alpha}{\pi}\right)^2 \mathrm{Re}\, F(s) = \left(\frac{\alpha}{\pi}\right)^2 \left[-c_3 L^3 + \tilde{c}_2 L^2 + \tilde{c}_1 L + \tilde{c}_0\right]$$

Comments

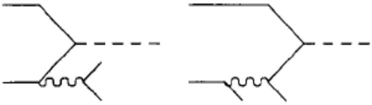
- The large L^3 contribution cancels in the sum $R + V$! It's a general result
- For a given set up, the correction is dominated by the lightest emitted pairs because of L appearance
- The correction can significantly depend on the total energy Δ of the emitted pair if it is "small" (Δ maximum energy below which pairs can not be observed or are included, "by definition", in the event sample)
- Beyond soft approximation, a $2 \to 4$ (MC) calculation of the real part is needed to account for experimental cuts realistically
- Hadrons require special care (R parameterization)

QED Structure Functions (SFs)

In the LL approximation (all L terms with each power of α), the photonic pair correction can be obtained by means of QED SFs. For single W production (final–state pairs only)

$$d\sigma(p+p \to W \to \nu + l(E)) \Longrightarrow \int_{1-\Delta}^{1} dx\, d\sigma(p+p \to \nu + l(xE)) D^{\text{pairs}}(x,s)\Theta(\text{cuts})$$

where $D^{\text{pairs}}(x,s)$ is known analytically (in the collinear limit) as sum of two different pair production mechanisms (virtual corrections through regularized splitting functions and running of α)



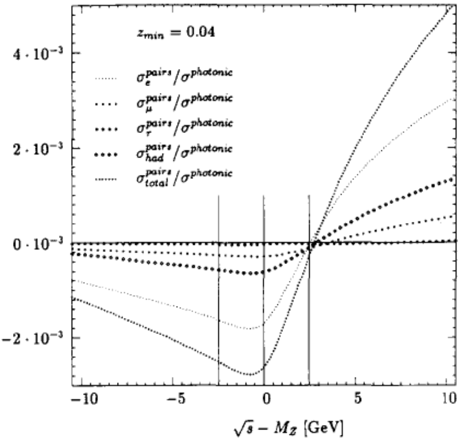
(a) (b)

$$D^{\text{pairs}}(x,s) = D^{\text{pairs}}_{\text{NS}}(x,s) + D^{\text{pairs}}_{\text{S}}(x,s)$$

1. $D^{\text{pairs}}_{\text{NS}}(x,s)$ [(a)] – All pair flavors. Dominant effect (soft pairs). Truncated $\mathcal{O}(\beta^2)$ and exponentiated $\mathcal{O}(\beta^\infty)$ solutions known, with β expansion parameter
2. $D^{\text{pairs}}_{\text{S}}(x,s)$ [(b)] – Only e^+e^- pair production. Infrared regular and finite, i.e. important for very high energy pairs only. $\mathcal{O}(\beta^2)$ (and $\mathcal{O}(\beta^3)$) solution(s) known

Typical size and impact of the correction

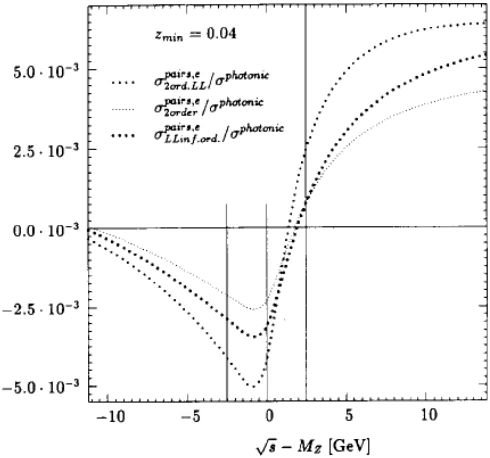
S. Jadach, M. Skrzypek and M. Martinez, Phys. Lett. **B280** (1992) 129



- At the Z^0 pole, where $L \sim \ln(M_Z^2/m_l^2)$ and $\Delta \sim \Gamma_Z/M_Z \Longrightarrow \delta_{\rm pair}^{\rm ISR} \approx -0.25\%$ $\Longrightarrow \Delta M_Z \approx -1$ MeV and $\Delta\Gamma_Z \approx -2$ MeV
- e^+e^- pairs are the very dominant contribution ($\approx 80\%$)

Correction hierarchy & Perturbative vs. SFs calculation

S. Jadach, M. Skrzypek and M. Martinez, Phys. Lett. **B280** (1992) 129



- Sub–leading (soft pair logs) and multiple pair contributions quite important
- $\mathcal{O}(\beta^\infty)$ $D^{\rm pairs}_{\rm NS}$ is an effective good approximation of the $\mathcal{O}(\alpha^2)$ result

What we are doing & questions

1. We are going to implement (QED) pair corrections through SFs in HORACE and evaluate the corresponding W mass shift. We plan to study the uncertainty induced by different SFs formulae. [comparisons with results by Andrej Arbuzov?]
2. Full perturbative calculation of pair corrections to $\bar{u}d \to W \to l + \nu$ tricky! (two–loop e.w. calculation). Simplification: we are computing the soft real + virtual pair corrections to the W leptonic decay rate.
3. We plan to cross–check the results of the two approaches, where comparable.

Questions

- Bare event selection for μ and Calo for e (like for photons)?
- Typical value for Δ at the Tevatron, i.e. maximum energy of the “lost” fermion pairs? [more inclusive the event selection, smaller the i.r. logs and the total correction]
- Same Δ for CDF and D0?
- (Treatment of high–invariant mass pairs from Z^0 splitting?)

Some references

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