

Teoria delle Interazioni Fondamentali

A. A. 2021/2022

For the preparation of the Teoria delle Interazioni Fondamentali exam you have to:

1. solve completely one of the following problems, with all the details;
2. study the content of the lectures and be ready to answer a few theoretical questions on the topics presented there.

1. **Electron-positron annihilation: production of a $\mu^+\mu^-$ pair**

Consider, in lowest order approximation, the process $e^+e^- \rightarrow \mu^+\mu^-$, neglecting the mass of the initial state electron (positron).

- (a) Write, first in QED and then in the full electroweak Standard Model, at tree level, the scattering amplitude of the process and then compute the unpolarised squared matrix elements.
- (b) Compute in the two models the total cross section, and discuss its dependence on the center-of-mass energy, in the energy range from 0 to 2000 GeV.
- (c) Compute the forward-backward asymmetry distribution, as a function of the invariant mass of the lepton pair. Give a qualitative discussion of the shape of the distribution, in the same invariant mass range of the previous point.
- (d) In QED, draw (the analytical expressions are not required): *a*) all the 1-loop Feynman diagrams which contribute to the process $e^+e^- \rightarrow \mu^+\mu^-$; *b*) all the tree-level diagrams which contribute to the process $e^+e^- \rightarrow \mu^+\mu^-\gamma$. Prepare an introduction to the problem of the UV renormalisation and to the problem of the cancellation of the IR divergences.
- (e) Compute the one-loop self-energy correction in QED, considering in the loop a fermion with generic mass m_f . Discuss its behaviour as a function of q^2 .

2. **Electron-positron annihilation: total cross-section into hadrons**

Consider the on-shell scattering amplitude for the decay of a virtual photon into a quark-antiquark pair $\gamma^* \rightarrow q\bar{q}$ (in the massless quark approximation).

- (a) Calculate the one-loop virtual QCD corrections in the *dimensional-regularization* scheme.
- (b) Calculate the tree-level squared matrix element for the real gluon emission process $\gamma^* \rightarrow q\bar{q}g$ in the *dimensional-regularization* scheme.
- (c) Compute the total cross-section at order α_S by integrating the real matrix element and adding together real and virtual corrections.

(See e.g. Chap. 2 of *Application of Perturbative QCD*, R. D. Field)