Teoria delle Interazioni Fondamentali

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Electroweak interaction

1. **Scattering $b\bar{b} \to \mu^+\mu^-$ in the SM**
   Compute in the SM, in lowest order approximation, for an arbitrary value of the center-of-mass energy $\sqrt{s}$, the total unpolarized cross section and the differential distribution $d\sigma/d\cos\theta$, where θ is the scattering angle of the μ^+ with respect to the direction of the incoming b quark, in the partonic center-of-mass system.

   - Compute the scattering amplitude keeping the full dependence on the masses of the b quark and of the muon.
   - Repeat the calculation neglecting the masses of the muon and, eventually, of both muons and b quarks.
   - In the fully massless approximation, compute the polarized cross sections, with the fermions projected on all the possible helicity states. Compare the result of the previous point (in the massless approximation) with the sum of all the polarized cross sections.
   - In the fully massive approximation, compute the forward-backward asymmetry defined as $A_{FB} = (\sigma_F - \sigma_B) \ / \ (\sigma_F + \sigma_B)$, where $\sigma_F = \int_0^1 d\cos\theta \, \frac{d\sigma}{d\cos\theta}$ and $\sigma_B = \int_{-1}^0 d\cos\theta \, \frac{d\sigma}{d\cos\theta}$. Discuss the impact of the fermion masses.

2. **Decay $t \to W^+ b$ in the SM**
   Compute the total rate of the decay of a top quark into a W^+ boson and a b quark.

   - Compute the unpolarized decay rate.
   - Compute the polarized decay rates for the different W boson polarizations. Check that the sum ot the polarized decay rates corresponds to the result of the previous point.
   - Compute the decay rate for a longitudinally polarized W boson by means of the equivalence theorem and discuss the accuracy of this approximation.

3. **High-$p_\perp$ W boson production**
   Compute in lowest order approximation the production of a W boson and one quark, focusing on the scattering process $qg \to qW$.

   - Compute the differential cross section for the production of one W of given transverse momentum $p_\perp$.
   - Assuming that the W boson is on-shell, discuss the kinematics of the massless fermion-antifermion pair the W may decay into, as a function of the W transverse momentum.

Perturbative QCD

1) Electron-positron annihilations: total cross-section into hadrons

Consider the on-shell scattering amplitude for the decay of a virtual photon into a quark-antiquark pair $\gamma^* \to q\bar{q}$ (in the massless quark approximation).

a) Calculate the one-loop virtual QCD corrections in the *dimensional-regularization* scheme.

b) Calculate the tree-level squared matrix element for the real gluon emission process $\gamma^* \to q\bar{q}g$ in the *dimensional-regularization* scheme.

c) Compute the total cross-section at order α_S by integrating the real matrix element and adding together real and virtual corrections.

(See e.g. Chap. 2 of *Application of Perturbative QCD*, R. D. Field)

2) The Parton Model

Consider the deep-inelastic lepton-hadron scattering (mediated by a virtual photon).

a) Decompose the square of the invariant amplitude in terms of leptonic and hadronic tensors. Write explicitly the tensors in terms of lepton momenta and structure functions.

b) Write the differential cross section in the laboratory frame. Consider separately the absorption of transverse and logitudinal photons.

c) Consider the deep-inelastic limit and show that, in the parton model, the structure functions directly measure the parton (quark) distribution functions.

(See e.g. Chap. 4 of *Application of Perturbative QCD*, R. D. Field)

3) Altarelli-Parisi splitting functions

Consider the gluon emission contribution to the (parton level) deep-inelastic scattering: $\gamma^* q \to qg$.

a) By using a *physical gauge* and power counting show that intereference diagrams do not give rise to collinear singularities.

b) By using the Sudakov parameterization consider the contribution given by the *ladder* diagram (with a single cell) for the $q \to q(z) + g(1-z)$ splitting in the collinear limit. From the previous result, extract the Altarelli-Parisi splitting function $P_{qq}^{(0)}(z)$ for $z \neq 1$.

c) By using the probability conservation constraint, derive the endpoint $z = 1$ limit of the splitting function $P_{qq}^{(0)}(z)$.

(See Chap. 4 of *QCD and Collider Physics*, R. K. Ellis, W. J. Stirling & B. R. Webber, and/or Chap. 1 of *Basics of Perturbative QCD*, Y. L. Dokshitzer, V. A. Khoze. A. H. Mueller, S. I. Troyan, in particular Problems 1.2, 1.3, 1.4)