Teoria delle Interazioni Fondamentali

A. A. 2016/2017

Solve in detail one problem of the first and one of the second group.
Study the theoretical background and prepare the discussion of all the others.

QED and electroweak physics: Exercises

1. Compute the $\mathcal{O}(\alpha)$ coefficient of the QED β function.

   - Compute in dimensional regularization the self-energy corrections to the photon vacuum polarization, focusing on the UV divergent part.
   - Compute the renormalized electric charge and identify its dependence on the renormalization scale μ.
   - Derive the expression of the β function.

2. Compute at tree level in QED the cross sections of the reaction $e^-e^- \to e^-e^-$ (Moeller scattering).

   - Compute the unpolarized squared matrix elements, including the mass of the electrons.
   - Compute the differential cross section with respect to the scattering angle θ defined in the system center-of-mass reference frame.
   - Discuss the properties of the total cross section.

3. Compute at tree level in QED the cross sections of the reaction $e^-\mu^- \to e^-\mu^-$.

   - Discuss the differences with respect to the results of the previous point.
   - Discuss the relation between this process and the process $e^+e^- \to \mu^+\mu^-$.
   - Repeat the calculation in the full electroweak Standard Model. Discuss the changes of the differential cross section in presence of the contribution of a Z-boson exchange.

4. Compute at tree level in the full electroweak Standard Model the total decay rate of a muon, in the process $\mu^- \to \nu_\mu e^- \bar{\nu}_e$.

Perturbative QCD: Exercises

1) Electron-positron annihilations: total cross-section into hadrons

Consider the on-shell scattering amplitude for the decay of a virtual photon into a quark-antiquark pair $\gamma^* \to q\bar{q}$ (in the massless quark approximation).

a) Calculate the one-loop virtual QCD corrections in the *dimensional-regularization* scheme.

b) Calculate the tree-level squared matrix element for the real gluon emission process $\gamma^* \to q\bar{q}g$ in the *dimensional-regularization* scheme.

c) Compute the total cross-section at order α_S by integrating the real matrix element and adding together real and virtual corrections.

(See Chap. 2 of *Application of Perturbative QCD*, R. D. Field)

2) Initial-state collinear singularities

Consider the deep-inelastic lepton-hadron scattering at order α_S.

a) By using a *physical gauge* and power counting show that intereference diagrams do not give rise to collinear singularities.

b) By using the Sudakov parameterization consider the contribution given by the *ladder* diagram (with a single cell) for the $q \to q(z) + g(1-z)$ splitting in the collinear limit. Discuss the result in terms of the $P_{qq}^{(0)}(z)$ Altarelli-Parisi splitting function.

c) Consider the case of the $q \to g(z) + q(1-z)$ and of the $g \to q(z) + q(1-z)$ splitting.

(See Chap. 1 of *Basics of Perturbative QCD*, Y. L. Dokshitzer, V. A. Khoze. A. H. Mueller, S. I. Troyan, and in particular Problems 1.2, 1.3, 1.4)

3) Deep-inelastic lepton-hadron scattering

Consider the gluon emission contribution to the (parton level) deep-inelastic scattering: $\gamma^* q \to qg$.

a) By analogy with the Compton scattering in QED derive the transverse momentum p_T distribution of the outgoing quark in the limit of small p_T.

b) From the previous result, extract the Altarelli-Parisi splitting function $P_{qq}^{(0)}(z)$ for $z \neq 1$.

c) By using the probability conservation constraint, derive the endpoint $z = 1$ limit of the splitting function $P_{qq}^{(0)}(z)$.

(See Chap. 4 of *QCD and Collider Physics*, R. K. Ellis, W. J. Stirling & B. R. Webber, and/or Chap. 10 of *Quarks and Leptons*, F. Halzen and A. D. Martin.)