Higher orders and resummations for precision physics

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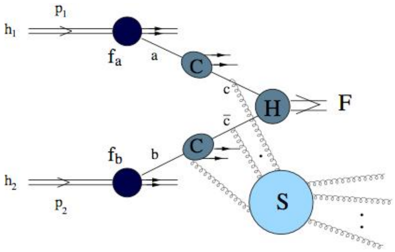
Hadronic cross sections in perturbative QCD



- h_1, h_2 = initial state hadrons (with momenta p_1, p_2)
- f_a, f_b = parton distribution functions
- C = coefficient functions (partonic splitting)
- H = perturbatively computed partonic event
- F = final state particle(s)
- S = resummation of soft radiation from incoming partons

$\longrightarrow$ Precise predictions depend on good knowledge of f,C,H *and* S!

Inclusive QCD hard scattering

$$h_1(p_1) + h_2(p_2) \to F(Q) + X$$

F = final-state system of high invariant-mass Q (jets,vector bosons, heavy quarks, Higgs), X = unobserved

- QCD approach is based on *factorization theorems*:
  - long distance (hadronic, M_{had}) physics
  - short distance (partonic, $Q \gg M_{had}$) physics
  - Factorization is not exact but corrections are $\mathcal{O}(M_{had}/Q)$
- $\sigma_{had}(p_1, p_2) \sim \int\int dx_1 dx_2 f_{a/h_1}(x_1, \mu_F) f_{b/h_2}(x_2, \mu_F) \sigma_{ab}^{part}(x_1 p_1, x_2 p_2, \mu_R, \mu_F)$
- QCD predictions require:
  - *Specific (process-dependent) theoretical calculations* (σ_{ab}^{part}) computable as a perturbative series in the QCD coupling $\alpha_S(\mu_R)$: LO (just order of magnitude), NLO (non trivial, today's standard), NNLO (today's frontier), ...
  - *Universal (process-independent) inputs*, primarily the coupling and the parton distribution functions (pdf) $f_a(x_1, \mu_F)$
- Main features of perturbative QCD:
  - Asymptotic freedom (α_S large/small at low/high Q)
  - Pdf scale evolution ($f(x, Q)$) predictable/computable, once initial conditions ($f(x, Q_0)$) extracted from experiments

K-factor

- LO cross sections suffer from large scale uncertainties: σ^{part} does not depend on $\mu_R, \mu_F \rightarrow$ *pdfs and* α_S *dependence are not balanced*
- Reliable results start at NLO

$$K = \frac{\sigma_{HO}(pp \rightarrow H + X)}{\sigma_{LO}(pp \rightarrow H + X)}$$

- α_S and pdfs have to be consistently evaluated at HO and LO as well (otherwise K could be larger,since α_S(NLO) < α_S(LO))
- Partonic cross sections known up to NNLO
  AP functions recently computed to 3-loops
  $\rightarrow$compute *full NNLO K-factors*

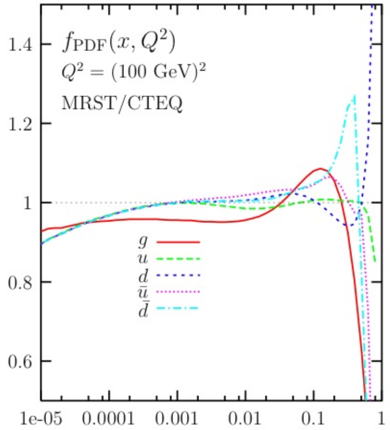
Scale dependence

- Usually one fixes a ”natural” scale μ_0 (typically the one that allows to absorb large logarithms...)
- Then μ_R, μ_F are independently or collectively varied within

$$\frac{\mu_0}{a} \leq \mu_F, \mu_R \leq \mu_0 a$$

- Dependence on $\mu_R, \mu_F \rightarrow$ evaluation of theoretical uncertainty ?
  - $\rightarrow$ The narrower the uncertainty band is, the smaller the HO corrections are expected to be (not always true!)
  - $\rightarrow$ In principle the scale uncertainty should be reduced when going to higher orders (not always true!)
  - $\rightarrow$ BUT remember that all this is unphysical and there is no rigorous way to estimate the theoretical uncertainty other than performing the higher-order calculation!

Parton Distribution Functions



- Differences between pdfs arise from
  - $\rightarrow$ choice of data points
  - $\rightarrow$ theoretical assumptions made for the fit
  - $\rightarrow$ choice of tolerance used to define the error in the fit
- Low-x ($x<10^{-3}$) and high-x ($x>0.7$) regions are critical: uncertainties of a few tens of %
- Intermediate-x region more reliable: uncertainties of a few %
- No clear separation between regions in the gluon case

Next challenges at colliders

- Precision QCD
  - H,W,Z and heavy quark hadroproduction
    - $\longrightarrow$ measured with high experimental accuracy
  - Multiparton final states
    - $\longrightarrow$ background to SUSY, UED, . . .
    - $\longrightarrow$ measurement of couplings
- LO is not enough
  - Large renormalization scale uncertainty (α_S scale not defined)
  - Large factorization scale uncertainty
  - Large corrections from higher orders
  - Jet structure appears only beyond LO
  - $\longrightarrow$ Reliable predictions only at NLO
  - $\longrightarrow$ Reliable estimate of errors only at NNLO
  - $\longrightarrow$ Resummation necessary in some region of the phase space

State of the Art - at a glance

Relative Order	$2 \to 1$	$2 \to 2$	$2 \to 3$	$2 \to 4$	$2 \to 5$	$2 \to 6$
1	LO					
α_s	NLO	LO				
α_s^2	NNLO	NLO	LO			
α_s^3	NNNLO	NNLO	NLO	LO		
α_s^4				NLO	LO	
α_s^5					NLO	LO

- LO Automated and under control, even for multiparticle final states
- NLO Well understood for $2 \to 1$ and $2 \to 2$ in SM and beyond
- NLO Many new $2 \to 3$ calculations from Les Houches wish list since 2007
- NLO Very first $2 \to 4$ LHC cross section in 2008 $q\bar{q} \to t\bar{t}b\bar{b}$
- NLO Important developments in automation, $W + 3$ jets (2009)
- NNLO Inclusive and exclusive Drell-Yan and Higgs cross sections
- NNLO $e^+e^- \to 3$ jets, but still waiting for $pp \to \mathrm{jets},\ W + \mathrm{jet},\ t\bar{t}, VV$
- NNNLO F_2, F_3 and form-factors

NLO Automation

- Combination of infrared divergent parts (dipole subtraction) has become standard and automated

[Gleisberg,Krauss(SHERPA);Frederix,Gehrmann,Greiner(MadGraph) Seymour,Tevlin(TevJet)Hasegawa,Moch,Uwer]

- One-loop matrix elements: major breakthroughs

Unitarity Methods

Use unitarity cuts on loop diagrams to compute tensor coefficients as products of tree amplitudes

[Bern,Dixon,Dunbar,Kosower(94); Britto,Cachazo,Feng(04); Berger,Bern,Dixon,Forde,Kosower(06); Giele,Kunzst,Melnikov(08)]

OPP Method

New reduction formalism for tensor integrals: reduce 1-loop amplitudes to scalar integrals at the integrand level

[Ossola,Papadopoulos,Pittau(06)]

implemented in **BlackHat, Helac/CutTools, Rucola**

Available codes

- **Rocket** [Giele,Zanderighi(08)]
  - up to 1-loop 20 gluon amplitudes! [Giele,Zanderighi(08)]
  - NLO W+3j cross section [Ellis,Melnikov,Zanderighi(08)]
  - NLO WW+2j cross section [Melia,Melnikov,Rontsch,Zanderighi(10)]
  - NLO e+e- —>5j cross section [Frederix,Frixione,,Melnikov,Zanderighi(10)]
- **BlackHat** [Berger et al.]
  - 1-loop 8 gluon amplitudes
  - 1-loop W+5j amplitudes (08)
  - NLO W+3j and Z+3j cross section (09,10)
  - NLO W+4j cross section (10)
- **Helac/CutTools** [Cafarella et al.(09)]
  - 1-loop amplitudes for $q\bar{q}, gg \to t\bar{t}b\bar{b}, b\bar{b}b\bar{b}, W^+W^-b\bar{b}, t\bar{t}gg, Wggg, Zggg$
  - NLO $pp \to t\bar{t}b\bar{b}$ cross section

  [Bevilacqua,Czakon,Papadopoulos,Pittau,Worek(09)]
  [see also Bredenstein,Denner,Dittmaier,Pozzorini(09)]
- Goal at NLO: all $2 \to 4(5,6)$ processes with Unitarity/OPP methods

NLO Matching

Parton Shower Generator	Matrix Element Generator
Resums leading logs to all orders	Only go up to NLO
High multiplicity *hadrons* in final state	Low multiplicity *partons* in final state
Good for regions of low relative p_T	Good for regions of high relative p_T
Total rate accurate to LO	Total rate accurate to NLO

The perfect matching

- generates total rates accurate at NLO
- treats hard emission as in Matrix Element Generators
- treats soft/collinear emission as in Parton Shower Generators
- generates a set of fully exclusive events which can be interfaced with a hadronization model

NLO Matching

- **MC@NLO** [Frixione,Webber(02)]
  - add difference between exact(ME) NLO and approx.(PS) NLO
  - automatization (aMC@NLO) based on FKS subtraction @ NLO

    [Frederix,Frixione,Maltoni,Stelzer(09)]

  → dependent on the shower details

  → difference may be negative
- **POWHEG** [Nason(04)]
  - Generate the hardest emission at NLO accuracy (mod. Sudakov)
  - Angular-ordered showers: add truncated shower from hard scale

  → always positive weights

  → discrepancies with respect to MC@NLO thoroughly explained in several publications

Ingredients for NNLO

- For a general $2 \rightarrow n$ process we need
  - Two-loop amplitude for $2 \rightarrow n$
  - One-loop amplitude for $2 \rightarrow n+1$
  - Tree-level amplitude for $2 \rightarrow n+2$
- Each term has its own singularities
  - Ultraviolet (removed by renormalization)
  - Infrared (have to cancel among each other)

$\longrightarrow$ Much more difficult than NLO cancellation!

Cancellation of singularities

- Fully inclusive quantities
  - analytical computation of contributions is possible
  - explicit cancellation of singularities
  - → DIS [Zijlstra,van Neerven(92)]
  - → Single Hadron [Rijken,vanNeerven(97);Mitov,Moch(06)]
  - → DY [Hamberg,van Neerven,Matsuura(91)]
  - → H [Harlander,Kilgore(02);Anastasiou,Melnikov(02);Ravindran,Smith,van Neerven(03)]
- Fully exclusive quantities (real world!)
  - IR singularity structure at NNLO understood
    [Catani,Grazzini;Campbell,Glover;Bern,DelDuca,Kilgore,Schmidt; Kosower,Uwer;Sterman,Tejeda-Yeomans]
  - numerical integration still very difficult
  - → **Sector Decomposition**
  - → **Subtraction Method**

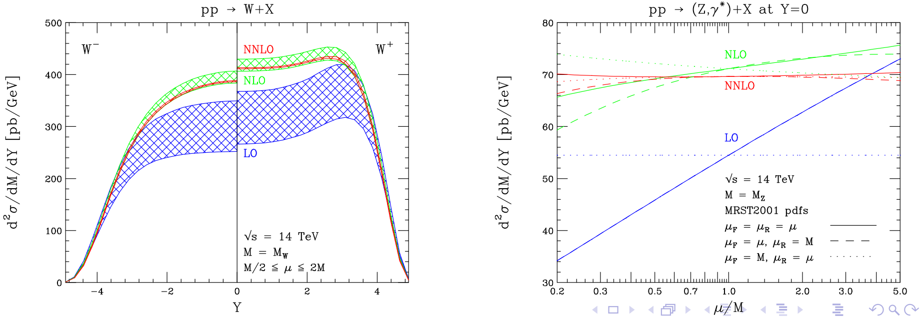
Sector Decomposition

"Split the integration region into sectors, each containing a single singularity, and explicit the pole by expanding it into distributions"

Binoth,Heinrich[00,04];Anastasiou,Melnikov,Petriello[04]

AMP developed a fully automated procedure to compute pole coefficients and finite terms and applied it to

$$\text{H/W/Z(04), QED } \mu\text{-decay(05), } b \to c l \bar{\nu}_l \text{(08)}$$



Subtraction Method

"Add and subtract a local counterterm with the same singularity structure of the real contribution that can be integrated analytically over the phase space of the unresolved parton"

NLO:Ellis,Ross,Terrano[81];Frixione,Kunzst,Signer[95];Catani,Seymour[96]
(NNLO):Kosower[03,05];Weinzierl[03];Frixione,Grazzini[04]
Gehrmann,Glover[05];Somogyi,Trocsanyi,DelDuca[05,07]

$$
\begin{aligned}
d\sigma &= \int_{n+1} r d\Phi_{n+1} + \int_n v d\Phi_n \\
d\sigma &= \int_{n+1} (r d\Phi_{n+1} - \tilde{r} d\tilde{\Phi}_{n+1}) + \int_{n+1} \tilde{r} d\tilde{\Phi}_{n+1} + \int_n v d\Phi_n
\end{aligned}
$$

The *Antenna Subtraction Method* developed by A and T. Gehrmann and Glover has been used for the NNLO QCD calculation of

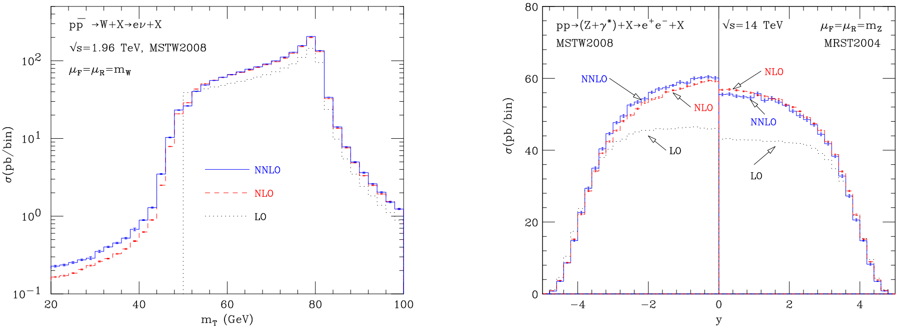
$$e^+e^- \to 3 \text{ jets}$$

A.Gehrmann,T.Gehrmann,Glover,Heinrich[07]

Subtraction Method

NNLO subtraction has been applied also to Higgs and Vector Boson production at the LHC

H:Catani,Grazzini[07];W,Z:Catani,Cieri,DeFlorian,Ferrera,Grazzini[09]



- *Z*: result changes with different sets of pdfs
- *W*: large NNLO effects at low m_T, instabilities at $m_T \sim 50$ GeV

The need for resummation

Partonic cross section as a perturbative series

$$
\begin{aligned}
\sigma_{ab}^{part}(p_1, p_2, Q, Q_i, \mu_R, \mu_F) &= \alpha_s^k(\mu_R)[\sigma_{LO}(p_1, p_2, Q, Q_i) \\
&+ \alpha_s(\mu_R)\sigma_{NLO}(p_1, p_2, Q, Q_i, \mu_R, \mu_F) \\
&+ \alpha_s^2(\mu_R)\sigma_{NNLO}(p_1, p_2, Q, Q_i, \mu_R, \mu_F) + \ldots]
\end{aligned}
$$

- The fixed-order result gives reliable result only when all the scales are of the same order of magnitude
- If $Q_i \gg Q$ or $Q_i \ll Q$, the appearance of $\alpha_s log(Qi/Q)$ terms could spoil the perturbative result: they need to be resummed!

Resummation: well-known examples

- $\log(Q/Q_0)$
  - evolution of pdfs from input scale Q_0 to hard scale Q
  - collinear radiation from colliding partons: single logs
  - systematically resummed by DGLAP equation
- $\log(Q/\sqrt{S})$
  - hadronic c.m. energy $\sqrt{S}$ much larger than hard scale Q
  - multiple radiation over wide rapidity range: single logs
  - systematically resummed by BFKL equation
- $\log(Q^2/q_T^2)$
  - systems with invariant-mass $Q \gg q_T$
  - soft and collinear gluon emission: single and double logs
  - treated by means of soft-gluon resummation
- $\log(1 - Q^2/S)$
  - hadronic c.m. energy $\sqrt{S}$ comparable to hard scale Q
  - soft and collinear gluon emission: single and double logs
  - treated by means of soft-gluon resummation

Resummation: the main idea

$\alpha_s L^2$	$\alpha_s L$	$\dots$	$\dots$	$\mathcal{O}(\alpha_s)$	(LO)
$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	$\mathcal{O}(\alpha_s^2)$	(NLO)
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
$\alpha_s^n L^{2n}$	$\alpha_s^n L^{2n-1}$	$\alpha_s^n L^{2n-2}$	$\dots$	$\mathcal{O}(\alpha_s^n)$	$(N^n LO)$
LL	NLL	NNLL	$\dots$	$\dots$	

- Ratio of two successive rows: $\mathcal{O}(\alpha_s L^2)$
- improved expansion
  - *reorganization* of the terms into *towers of logs*
  - *all-order summation* of the terms in each class
- key-point: *exponentiation*

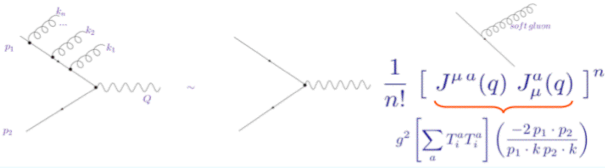
$$\sigma^{res} \sim \exp\left[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \dots\right]$$

- Ratio of two successive columns: $\mathcal{O}(1/L)$

Exponentiation

The observable must fulfill factorization properties both for

- dynamics (matrix element)
  - $\longrightarrow$ in the soft limit, multigluon amplitudes fulfill *generalized factorization formulae* given in terms of *single gluon emission probability*



- kinematics (phase space)
  - $\longrightarrow$ usually factorizable working in *conjugate space*

$$\delta^{(2)}(q_T - q_{T1} - \cdots - q_{Tn}) \quad = \quad \int d^2b\, e^{ib\cdot q_T}\, \Pi_i\, e^{ib\cdot q_T}$$

$$\log(Q^2/q_T^2) \quad \rightarrow \quad \log(Q^2 b^2)$$

$\longrightarrow$ generalized exponentiation of single gluon emission

[Collins,Soper,Sterman(1985)]

Matching with fixed-order

The resummed result has to be properly matched with the fixed-order calculation to avoid double counting

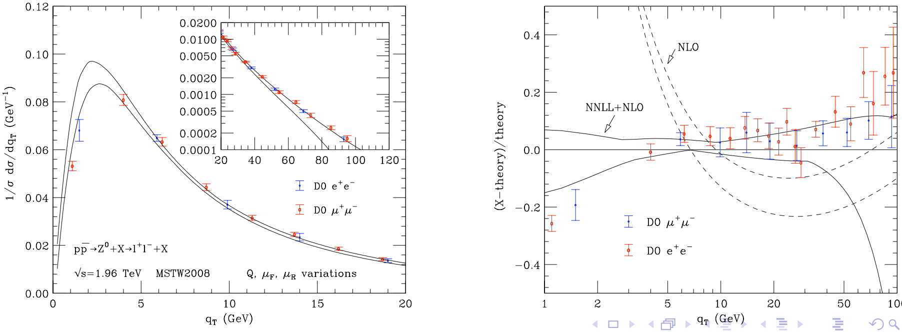
$$\sigma = \sigma^{res} + \sigma^{fix} - \sigma^{asym}$$

where σ^{asym} = expansion of resummed result to same order

- $q_T \ll Q$: $\sigma^{fix} \sim \sigma^{asym} \rightarrow \sigma = \sigma^{res}$
- $q_T > Q$: $\sigma^{res} \sim \sigma^{asym} \rightarrow \sigma = \sigma^{fix}$
- intermediate q_T: matching $\rightarrow \sigma$

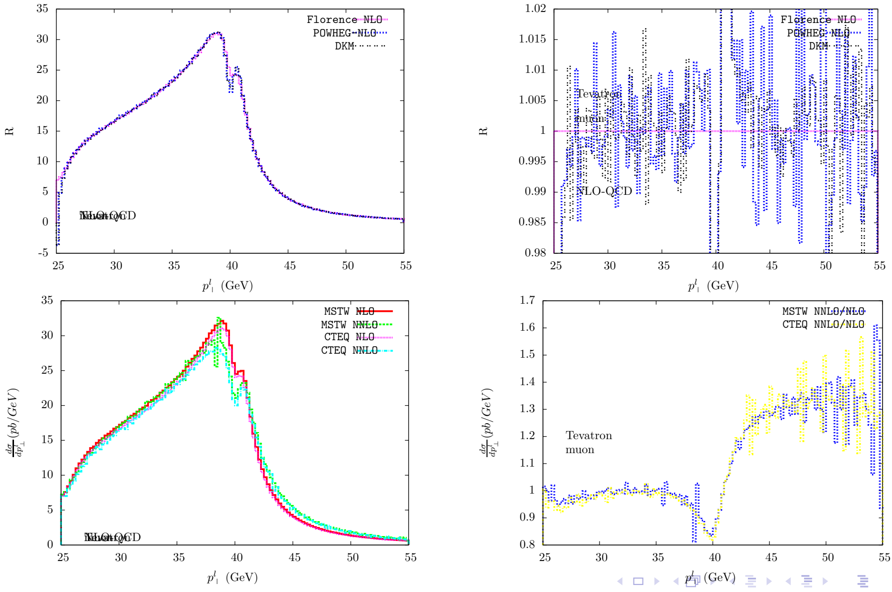
Drell-Yan at NNLL+NLO [Bozzi,Catani,deFlorian,Ferrera,Grazzini(10)]

- Normalized q_T distribution
- Scales fixed to Z mass
- → Uncertainty dominated by Q variation
- → Good agreement with Run II D0 data
- → Experimental errors are smaller than theoretical uncertainty
- most accurate QCD perturbative prediction for W and Z



And now some plots...

W production - Lepton Transverse Momentum



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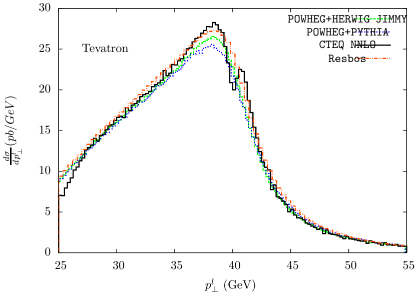
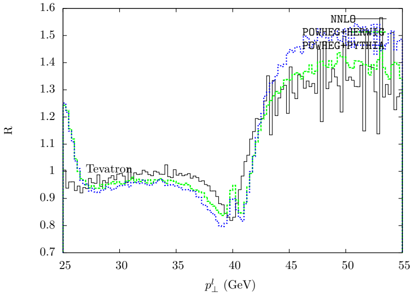
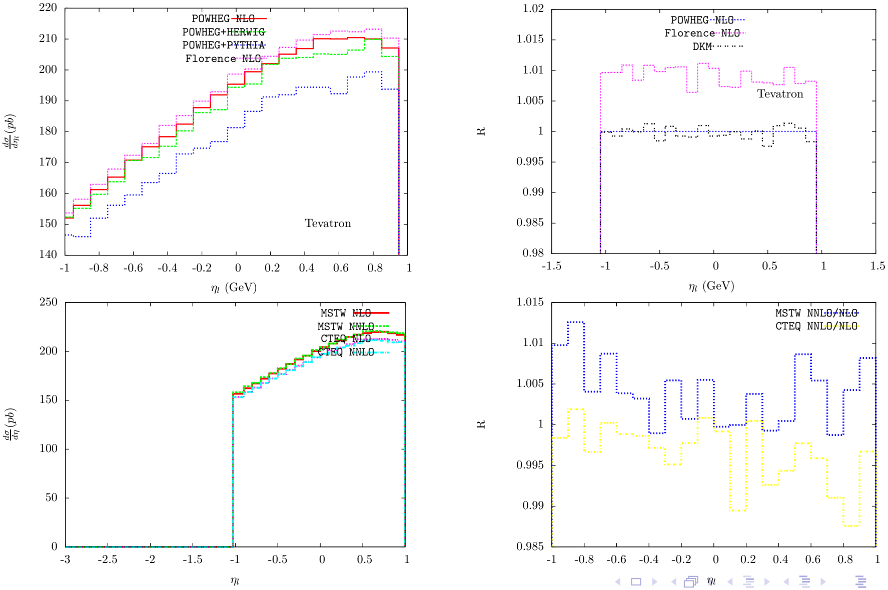




W production - Lepton Rapidity



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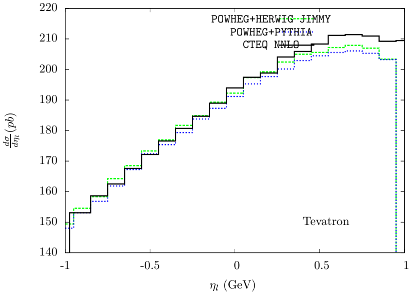
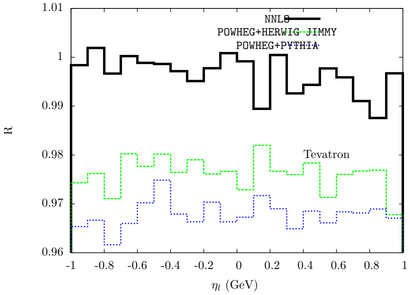
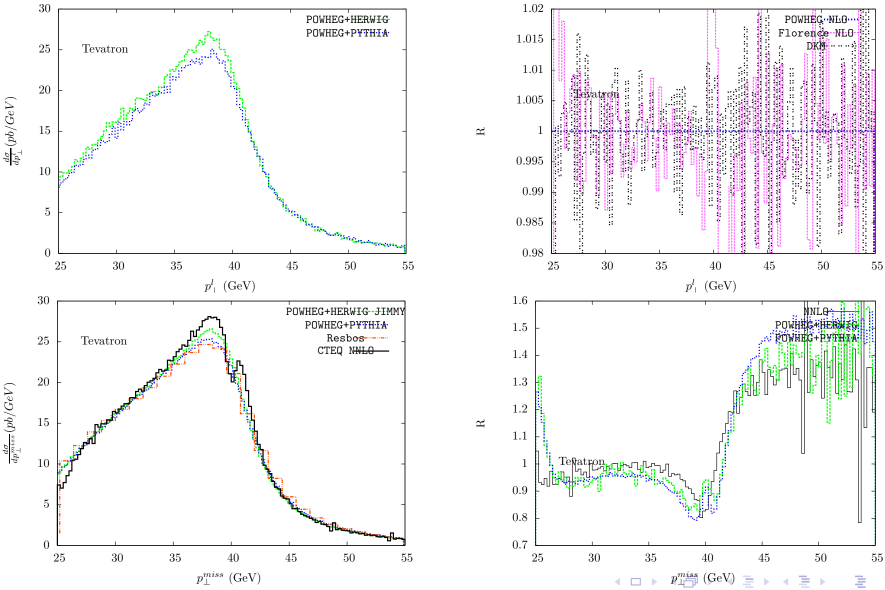
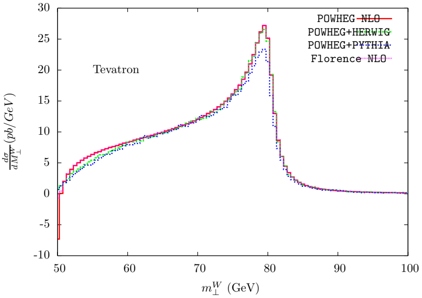


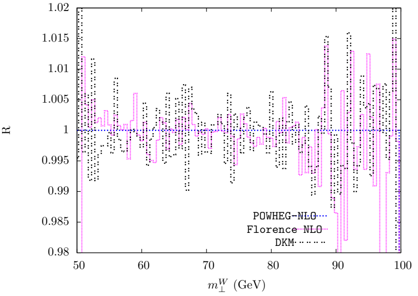


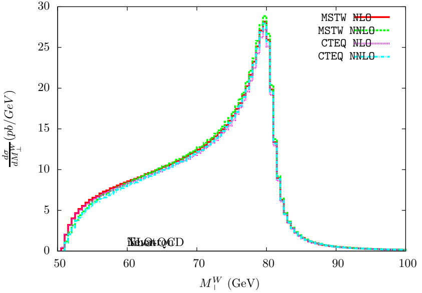
W production - Missing Transverse Momentum

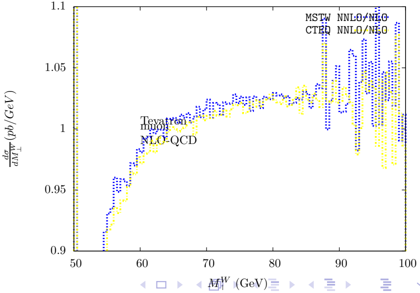


W production - Transverse Mass

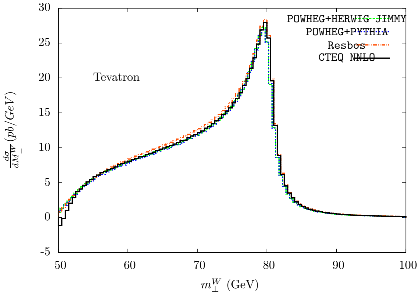


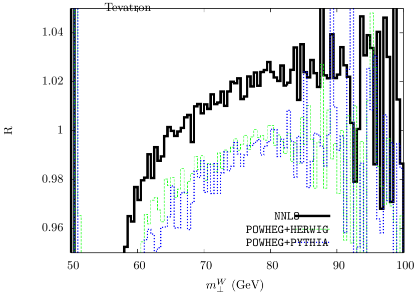




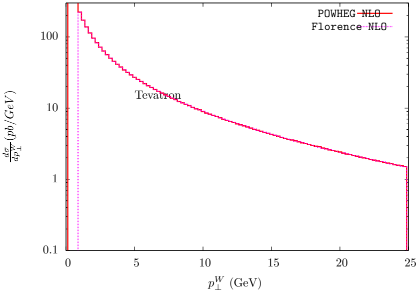


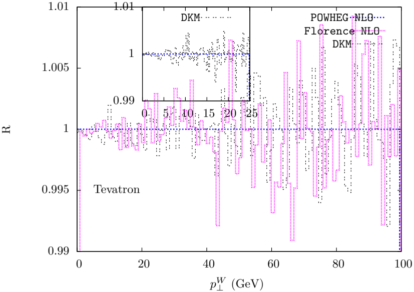
W production - Transverse Mass

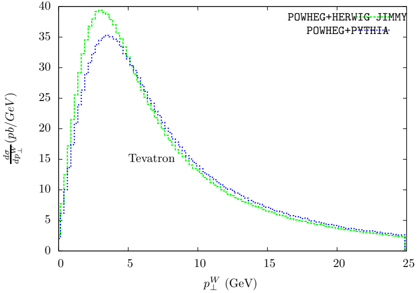


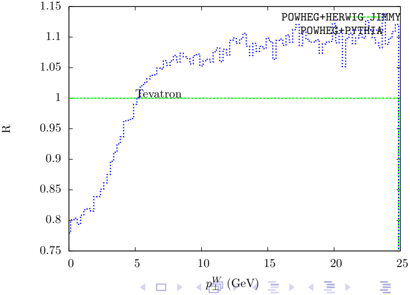


W production - W Transverse Momentum

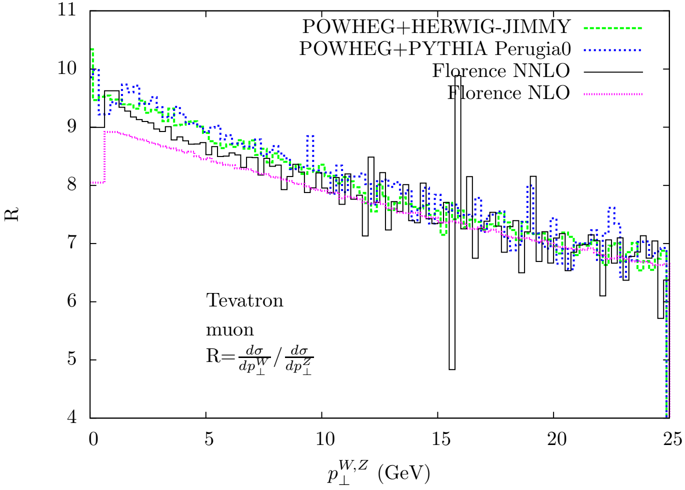








Ratio $p_{T,W}/p_{T,Z}$



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- authors of the different codes for providing numerical results
- Milano PC Farm for providing enough CPU-power
- Naperville Starbucks for providing
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  - → comfortable sofa
  - → relaxing music
  - → crazy amount of caffeine

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